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## Discourse-based Instructional Strategies to Engage Middle School Students With Standards for Mathematical Practice

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# Discourse-based Instructional Strategies to Engage Middle School Students With Standards for Mathematical Practice

Sandra L. Cookson

*Concordia University - Portland*

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College of Education

Doctorate of Education Program

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DISCOURSE-BASED INSTRUCTIONAL STRATEGIES TO ENGAGE MIDDLE  
SCHOOL STUDENTS WITH STANDARDS FOR MATHEMATICAL

Sandra Cookson, Ed.D.

Concordia University – Portland

College of Education

Dissertation submitted to the Faculty of the College of Education in partial fulfillment of the  
requirements for the degree of Doctor of Education in Teacher Leadership

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2017

## Abstract

Student achievement in mathematics is correlated with factors related to student engagement. Improving engagement has the potential to improve student achievement at the middle school level. The Common Core State Standards for Mathematics explicate eight specific Standards for Mathematical Practice (SMPs) that clarify the types of skills and learning dispositions associated with mathematical proficiency. The CCSS further urge teachers to engage students through pedagogical practices that provide opportunities to use the SMPs in increasingly complex ways. This study aims to identify how discourse is used as an instructional strategy to engage middle school mathematics students with the SMPs. Data was collected through a qualitative case study of a middle school mathematics teachers teaching five classes of mathematics to students at three grade levels. Instructional activities should be thoughtfully planned to emphasize independence and perseverance. A delicate balance of independent work and group interactions can support these dual goals. While discourse provides an opportunity to monitor students' engagement with many of the SMPs, thoughtfully planned activities and questioning routines help to guide the discussions toward the intended learning target.

*Keywords:* discourse, mathematics, middle school, Standards for Mathematical Practice

## Dedication

To Alexandra Lee and Andrew Craig. You both inspire me everyday by your very existence.

Remember, you can do anything.

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## **Chapter 1: Introduction**

Each year, states and school systems throughout the United States spend large sums of money to improve student achievement. Federal Title IIA funds have been designated to states for the purpose of improving student achievement by directly improving teacher quality. From 2012 to 2014, the United States Department of Education allocated approximately \$2.3 billion each year for the purpose of improving teacher quality (U.S. DOE, 2014b). In addition to recruitment and retention efforts, funding linked to Improving Teacher Quality generally target professional development activities and increasing the effectiveness of teachers. Professional development has taken many different forms, ranging from short-term workshops or conferences to longer term, more intensive course work or degree programs. As teachers learned new strategies for teaching and supporting learners, each teacher made choices about how to enact new knowledge and awareness into their classroom practice.

Since the adoption of the Common Core State Standards for Mathematics in 2010, the Standards for Mathematical Practice have helped to define habits and characteristics of high quality instruction (NCTM, 2014; O'Connell & SanGiovanni, 2014). Following an enactivist approach, the purpose of the proposed qualitative case study was to identify discourse-based structures and practices used by a middle school mathematics teacher to support student engagement with the eight Standards for Mathematical Practice (SMPs). The enactivist perspective recognizes that involving the teacher directly in a process of reflection and analysis allowed the teacher to observe patterns of interactions over time which bring awareness of processes and practices that impact student learning (Brown & Coles, 2012). This approach allowed the researcher to observe and collect data without directly impacting the choices of the teacher.

The over-arching question addressed in this study was, *how was discourse used as an instructional strategy to engage middle school students with the Standards for Mathematical Practices (SMPs)?* A qualitative case study was used to observe a teacher during lessons in a variety of middle school mathematics classes. Following classroom observations, interviews were used to probe deeper into the intended purpose of teacher-centered behaviors related to the use of discourse. An ongoing iterative cycle of data collection and member-checking was employed throughout the study. Data collection addressed the following three questions to inform the over-arching research question.

1. How were norms, routines, and classrooms expectations established and reinforced to support student discourse?
2. How were Standards for Mathematics Practice emphasized during instruction?
3. How were teacher-centered instructional strategies implemented to engage students with discourse around SMPs?

With the prominent references to SMPs throughout the Common Core Standards for Mathematics, it seemed reasonable to expect that instructional strategies would be specifically targeted toward promoting SMPs (making sense, reasoning, critiquing reasoning of others, modeling, etc.). Publications written for mathematics teachers stressed the importance of utilizing student discourse as a means for practicing the skills and dispositions outlined in the SMPs (e.g., Edwards & Townsend, 2012; Stephan, 2014; Thomas, Fisher, Jong, Schack, Krause, & Kasten, 2015). While studies in the field of mathematics suggest that the teacher and his/her pedagogical practices are an important component of student engagement and learning, such studies also conclude that more research is needed to identify the pedagogical choices

implemented as teachers attempt to engage students with mathematics (Attard, 2013; Darragh, 2013; Way, Reece, Bobis, Anderson, & Martin, 2015).

### **Background of the Problem**

Data from national (United States) and international testing showed a consistent pattern of declining student achievement in the field of mathematics as students progressed from elementary to middle school and the trend continued through high school (Lewis, 2013; Nation's Report Card, 2015; NCTM, 2014). Similarly, student engagement in mathematics followed a similar pattern as measured by decreased participation, more negative attitude, greater anxiety, and less confidence (Hannula, 2012; Way et al., 2015; OECD, 2014). While there was a positive correlation between mathematics engagement and achievement, no causal relationship had been definitively proven and little evidence existed to describe the teacher-centered behaviors that supported student engagement.

Findings from recent empirical studies in the field of mathematics suggest that the relationship fostered between students and teachers has a significant impact on student achievement (Attard, 2013; Walshaw, 2013). Effective pedagogical practices enacted by a noticing teacher have great potential for contributing to a quality learning environment and mathematical outcomes (Reznitskaya, Glina, Carolan, Michaud, Rogers, and Sequeira, 2012; Walshaw, 2013). In the socially bound context of the mathematics classroom, discourse is a vehicle through which learning is mediated (Mason, Drury, & Bills, 2007) and students can be supported as they confront disequilibrium (Kazak, Wegerif, & Fujita, 2015). In short, findings from previous studies suggest that a teacher who is able to integrate meaningful discourse as a feature of instructional practice will be better situated to improve student engagement.

Current emphases in other content areas supported the implementation of student discourse as a means for supporting student achievement and higher order thinking about content. Common Core Standards for English Language Arts outline expectations for speaking and listening at each grade level, kindergarten through grade 12. Similarly, Next Generation Science Standards (2013) identify proficiencies related to asking questions, interpreting data, engaging in argument, and communicating information. As professional development has been implemented to bring awareness to new expectations for communication in other content areas, these skills and pedagogical strategies seemed to be directly transferable to mathematics instruction. Yet, in research literature, “little attention appears to be given to the specifics of these pedagogical relationships” (Way, Reece, Bobis, Anderson, & Martin, 2015, pp. 629–630).

### **Statement of the Problem and Research Questions**

The eight Standards for Mathematical Practice identified in the Common Core State Standards for Mathematics (2010) outline key skills, processes, and habits “that mathematics educators at all levels should seek to develop in their students” (p. 6). These proficiencies were identified from a broad field of research about ways that mathematicians think and behave. Research about mathematics instruction also illuminated the myriad ways that teachers support student engagement with math – including emphasizing a growth mindset (Boaler, 2013), promoting risk-taking (Sharma, 2015), and providing hands-on explorations of mathematical concepts (Cheeseman, 2009). Research had yet to explore pedagogical choices for student engagement with Standards for Mathematical Practice, however. Bobis, Anderson, Martin, and Way (2011) noted that thoughtfully planned and monitored discourse-rich instructional practices support student identities that subsequently promote engagement and motivation in middle school mathematics courses.



Based on the importance of SMPs in mathematics instruction and the potential for discourse-rich instruction to improve student engagement, how can discourse be used as an instructional strategy to engage middle school student with the SMPs? Through classroom observations and interviews with a middle school mathematics teacher, the purpose of this study was to identify discourse-based structures and practices used by a middle school mathematics teacher to support student engagement with the eight Standards for Mathematical Practice (SMPs). This study was guided by the following questions:

1. How were norms, routines, and classrooms expectations established and reinforced to support student discourse?
2. How were Standards for Mathematics Practice emphasized during instruction?
3. How were teacher-centered instructional strategies implemented to engage students with discourse around SMPs?

The SMPs were important to this study as they describe the ways that students “increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise” (NGO & CCSSO, 2010, p. 8). Thus, the content of the mathematical engagement was defined in terms of the SMPs. The process under investigation was the discourse-based strategies used to engage students with the content. This study was about instructional practices enacted by a middle school mathematics teacher, not student responses to instruction. As such, data collection focused on the teacher and his choices, not students.

### **The Research Purpose**

As teachers engage in professional development, they learn new strategies, they are introduced to new concepts, and, with any luck, they are referred to research studies for more information about the conditions upon which theories and strategies were derived. However,

every classroom is different; student demographics are diverse, individual student needs vary, and the collective identity of the group changes from one class to the next. Teachers must constantly integrate, assess, and adapt content knowledge and pedagogical skills to meet the needs of the students sitting in the room at the moment.

The purpose of this study was to identify discourse-based structures and practices used by a middle school mathematics teacher to support student engagement with SMPs. In applying an enactivist approach to research on teacher practices, teacher reflection on factors influencing choice of instructional strategies was an important feature of the data collection. Frequent interviews with the teacher in the proposed study were planned. Interview question #3 (Appendix C) was specifically designed to elicit information that made the teacher's invisible, responsive processes visible and explicit to the researcher and future readers of the study. Hargreaves and Shirley (2012) identified reliance on evidence of student learning and responsive use of data to adjust instruction as characteristics of high-achieving schools. NCTM (2014) echoed the importance of responsive instruction based on the needs of students. As the teacher internally strategized on-the-fly to support student engagement with mathematical practices, it was important to understand how information collected through discourse provided data about current understanding, intended goals, and perceived learning tangles.

### **Rationale for Qualitative Research Methodology**

Merriam (2009) advocated for qualitative case study as useful to the investigation of “complex social units consisting of multiple variable of potential importance in understanding the phenomenon” (p. 50). In the proposed study, it was anticipated that the discourse-based structures enacted by the teacher might be dependent upon the needs and prior knowledge of students. These needs were unlikely to be visible to the researcher during classroom observations.

Semi-structured interviews were held following classroom observations to examine the factors influencing the teacher's choice of activities and discourse patterns.

This study focused on a specific phenomenon (discourse-based instructional practices used to enact Standards for Mathematical Practice) within the context of middle school classes taught by one teacher. Yin (2015) noted that quantitative case studies are useful when the boundaries between the phenomenon and the context are not clear. Discourse in a classroom is a highly contextual phenomenon in which conversational flow and questioning patterns are dependent upon each subsequent interaction or response. Although the teacher may have planned guiding questions at the start of the lesson, each student's point of access and background knowledge require a different line of questioning to appropriately scaffold instruction. As such, investigating instructional choices related to discourse and SMPs required consideration of the full context of the situation. Direct observations were implemented so that the researcher was able to witness discourse events in their context. Through observations, the researcher gained knowledge of specific incidents and behaviors “that can be used as reference points for subsequent interviews” (Merriam, 2009, p. 119).

## **Research Design**

Large-scale and small-scale qualitative research into student engagement with mathematics was conducted in New Zealand and Australia (Attard, 2013; Darragh, 2013; Way et al, 2015). These studies focused on students' perspectives of factors influencing their mathematics identities. In each study, it was noted that additional research was needed in the area of teachers' perceptions and pedagogical relationships fostered by teachers in the mathematics classroom. The current study addressed this gap.

This study was situated in a rural school in Central Maine. The teacher was responsible for providing mathematics instruction to students in grades six through eight. This arrangement is typical of many middle schools in rural Maine where a single teacher provides content-area instruction to students at multiple grade levels and often works with the same cohort of students for multiple consecutive years. While this study involved a single case of one teacher in a single school, there were also embedded cases of the teacher's different classes he interacted with throughout the day. The unique situations in the different classes were useful in identifying emerging patterns and situations that led to diverging behaviors and instructional strategies.

The purpose of the study was to identify discourse-based strategies used by the middle school mathematics teacher to engage students with Standards for Mathematical Practice. The teacher in the proposed study participated in professional development activities over the previous four years to increase his awareness of the SMPs and to improve his use of discourse-based pedagogy. Yet, it was anticipated that the ways in which a responsive teacher enacted instructional strategies might be highly dependent upon the needs of the students present in his classroom. By studying the embedded cases of this teacher's classes, educators and those who support them may be able to consider nuances of individual classrooms and students to make flexible choices about instructional practice to improve student engagement.

Between 2010 and 2013, the United States federal government spent an average of \$2.3 billion annually to improve teacher quality (U.S. DOE, 2014b). Maine's share of that funding averaged about \$10 million per year (Maine DOE, 2016). Additional local funds were also raised to support teacher quality. In Maine, efforts had been made over the over the same period of time to inform teachers about changes inherent in Common Core standards. In addition to content-area standards, such as those in mathematics, teachers were trained in cross-content connections

such as the connection between mathematics and English Language Arts (ELA) standards for speaking and listening. Rarely, however, were follow-up opportunities provided to determine whether new content and instructional strategies transferred to classroom practice. The proposed study was an opportunity to follow up on those trainings and to understand how one teacher enacted his training in the real-world situation of his classroom.

The focus on discourse not only connected what research had shown to be an important factor in student engagement, but discourse was viewed as something that any teacher could implement. The use of discourse in the classroom was not dependent upon an expensive program nor a specific textbook or set of resources. By understanding the factors influencing the use of discourse and connecting this instructional strategy to student engagement with SMPs, the researcher hoped to be able to identify factors that may lead to a positive impact on student achievement. It was beyond the scope of this study to determine what effect, if any, classroom discourse had on student achievement. The focus of this study was on discourse strategies used to engage students with SMPs.

The proposed qualitative case study was conducted through the use of observations, interviews, and document analysis. Data collection and analysis focused on discourse-based structures and activities related to SMPs. Due to the focus of the study on instructional practices enacted by the teacher, observations provided necessary data about practices used in various classes. Interviews were used to determine the desired intent of the practice and whether such intent was achieved. Observations and interviews together were used to analyze the ways that norms and expectations were implemented.

Twenty classroom observations were conducted over a 12 week period. Field notes were collected during each observation and observed classes were recorded for transcript analysis.

Recordings and field notes were used to identify discourse-practices initiated by the teacher (research question 3). Field notes also captured types of questioning strategies used - open-ended questions, analysis of scenarios that allow multiple solutions, eliciting student justifications of solutions, or providing opportunities for students to critique the reasonableness of others' solutions. Additionally, observations focused on the structure of feedback. Merriam (2009) pointed out that a benefit of observation is that it allows the researcher to see “things firsthand and use his or her own knowledge and expertise in interpreting what is observed rather than relying on once-removed accounts” (p. 119). As a participant observer, the researcher collected evidence of SMPs enacted through discourse.

### **Definition of Terms**

**Discourse.** Discourse was defined as “communication of thought by words; talk; conversation” (discourse, n.d.). Interactions between individuals through talk were considered discourse. Classroom discourse included episodes initiated by the teacher or student. The content of classroom discourse was not specified and did not refer solely to on-task verbal interactions. Much had been written about qualities of effective classroom discourse – e.g., how to facilitate discourse (Mercer & Sams, 2006), effective questioning strategies (NCTM, 2014), and assessing student knowledge through evidence collected during discourse (Marzano & Kendall, 2008). Truxaw, Gorgievski, and DeFranco (2008) defined mathematical discourse as “purposeful talk on a mathematics subject in which there are genuine contributions and interaction” (p. 58). While Wachira, Pourdavood, and Skitzki (2013) advanced a definition of mathematical discourse based on precise language. For the purpose of this study, classroom discourse referred to verbal interactions between students or between the teacher and student(s). Such verbal interactions

may have originated from physical models or written work, but the focus of the data collection was on utterances that were able to be captured auditorially.

**Instructional strategies.** Instructional strategies were the activities and processes enacted by the teacher for the purpose of conveying knowledge, skills, and academic habits. Ideally, teachers should utilize a variety of instructional strategies based on the goal of the lesson and knowledge of individual learners. Bobis, Anderson, Martin, and Way (2012) described instructional strategies as either student-centered or teacher-centered. “Teacher-centered strategies include worked examples, explication, demonstration, and structured questioning. Student-centered strategies include collaborative group work, practical tasks, problem solving, open tasks, investigation, games, and student presentations” (pp. 35-36). Although Bobis et al. differentiated between teacher and student-centered strategies, the decision to utilize student-centered strategies is still an instructional strategy chosen by the teacher. Therefore, any of these teacher-chosen instructional strategies were considered in this study.

**Standards for Mathematical Practice.** Standards for Mathematical Practice (SMPs) refer to the eight standards outlined in the Common Core State Standards for Mathematics (2010). The SMPs outline processes, proficiencies, and productive dispositions that “mathematics educators at all levels should seek to develop in their students” (p. 6). While the eight standards identify outcomes for students, in this study emphasis was given to teacher practices that support students in developing and refining the qualities identified in the eight standards. Appendix A lists the eight Standards for Mathematical Practice.

### **Delimitations, Limitations, and Assumptions**

**Delimitations.** Delimitations were implemented to focus the research project and provide boundaries for data collection. Perhaps the most significant delimitation was the choice to focus

this study on a single, embedded case. While other middle school mathematics teachers may have been available, the teacher in the proposed study was chosen because he worked with students across multiple grades. Within the context of his classroom, he worked to meet the needs of a range of students at multiple grade levels. While multiple teachers may have provided additional data, the case of this teacher was sufficiently broad to identify patterns of instructional strategies.

Another important delimitation was the focus on Standards for Mathematical Practice. Discourse in mathematics classrooms has been studied from many different perspectives: mindset (Boaler, 2013); student reflection on mathematical strategies (Coles & Scott, 2015); and embedded assessment (Hackenberg, 2010), to name a few. The Standards for Mathematical Practice identify key ways that mathematically proficient students think about and interact with mathematical concepts, yet a gap exists in the research linking discourse and teacher strategies to effectively engage students with SMPs. By focusing on the SMPs and discourse-based instructional strategies to engage students with SMPs, it was hoped that additional data would be added to the research field in this area.

Finally, the choice to study this topic through qualitative methodologies was another delimiter. In a similar study, Erickson examined the teacher-student interactions in classroom conversations from the perspective of music – examining timing, rhythm, and cadence of interactions (in Ravitch & Riggan, 2012). Both, Erickson's study and the present study relied on Vygotsky's theory of situated learning and pedagogical practices which support constructivist learning. While Erickson was more concerned with how interactions unfold, this study focused on the instructional strategies that emphasize dialogue. In the current study, an iterative process of data collection and member checking was utilized. From an enactivist perspective, the teacher



in the study played an integral role in enacting processes and making meaning of the data. Brown and Coles (2012) argued that through an enactivist approach, teachers observe patterns over time that bring awareness of processes and practices that impact student learning. As such, the enactivist approach results in the co-emergence of theories and allows the classroom teachers to make his/her own interpretations. The flexibility needed to engage the teacher in this way was only possible with a qualitative study.

**Limitations.** One significant limitation of this study was found in the demographics of the school. The school was situated in a rural community in Central Maine. The student population was 98% white and 100% English speaking. While these demographics were within the norm for small communities in Maine, they were not representative of the cultural diversity of the United States or the larger global education community. Expanding this study to other schools in the area would not have significantly altered the demographics.

Additionally, generalizability of findings from a particular case, especially one with limited demographic representation, may be seen as a limitation and potential threat to case study research. However, Flyvberg (2006) asserted that universal truths and applications in issues involving human affairs are not reliable. In social settings, he claimed, there are too many factors influencing outcomes to draw reliable generalizable conclusions. The case study approach is useful for understanding the role of multiple factors and situating the findings of the study within the full context of the setting. The proposed study relied on data collection from multiple classes taught by the same teacher. While research suggests that norms and classroom routines for discourse are important (e.g., Buchheister, Jackson, & Taylor, 2015; Leinwand, 2009), the reality of how such processes are enacted in the classroom may vary from class to class. Merriam

(2009) pointed out that “the general lies in the particular” (p. 51) and it is up to the reader of the research to decide whether the particular case is transferable to his/her situation.

**Assumptions.** The primary assumption in this study involved the participating teacher. It was assumed that he chose to participate because he was interested in improving his practice as a middle school mathematics teacher. His response when approached about the study was willingness and excitement. He expressed interest in having someone provide feedback about his instructional practice. During the preceding school year, he recorded several of his lessons and reviewed them independently as a means to reflect upon and improve his pedagogy.

Administrators at the school and district level described him as student-centered and reflective.

### **Summary**

As teachers learn new instructional strategies, they make choices about how to enact new knowledge and awareness into their classroom practice. Since the adoption of the Common Core State Standards for Mathematics in 2010, the Standards for Mathematical Practice have been a target for professional development to improve mathematics teaching and learning. Following an enactivist approach, the purpose of the proposed qualitative case study was to identify discourse-based structures and practices used by a middle school mathematics teacher to support student engagement with the eight Standards for Mathematical Practice (SMPs). By engaging the teacher in an iterative cycle of data collection and analysis, this case study captured “complex actions, perceptions, and interpretations” (Merriam, 2009, p. 44) related to discourse-based instructional strategies. Teachers and administrators who support them may benefit from this study as they consider the environment and interactions of their mathematics classrooms.

## Chapter 2: Literature Review

### Introduction to the Literature Review

This chapter establishes the connection between discourse-based instructional practices in middle school mathematics and student engagement and achievement. The *Background of the Problem* outlines evidence of declining engagement and achievement in middle school mathematics as a problem in both the United States and internationally (e.g., Nation's Report Card, 2015; NCTM, 2014; OECD, 2014). Targeted professional development to address new standards in other content areas revealed a common emphasis of engaging students through classroom discussions, group inquiry, and analysis of multiple perspectives. *Lessons Learned in Other Grades and Content Areas* presents some of the professional development activities used with educators in Maine. This section establishes credibility for this study's focus and highlights the emphasis on teacher involvement proposed in the *Conceptual Framework*.

Following the *Introduction*, *Background to the Problem*, and *Conceptual Framework*, the literature review begins with a look at the *Emphasis on Discourse as a Mathematics Pedagogy* as evidenced within resources targeting middle school mathematics teachers (e.g., Edwards & Townsend, 2012; Stephan, 2014; Suh & Seshaiyer, 2013). *Theories supporting the use of discourse* are considered as a means for judging the validity of pedagogical recommendations. Next, literature is presented to highlight the connection between discourse and student *Engagement in the Learning Process*. A closer look is taken at *Socio-cultural learning and engagement through discourse* as an important consideration for middle school aged students. Studies presented in this section will help establish a positive correlation between engagement and achievement while highlighting factors that influence student engagement (e.g., Attard, 2012; Brooks & Dixon, 2013; Darragh, 2013). The subsequent connection between discourse-

based practices and cognitive engagement with habits and processes expressed in the Common Core's Standards for Mathematical Practices is described in *Discourse for Higher Order Thinking and Cognitive Rigor*. The *Demand for cognitive rigor and the promise of challenging standards* seeks to draw the connection between literature outlining the shortcomings of the American education system and calls for reforms intended to address current problems (e.g., Darling-Hammond, 2010). Here too, the literature showed that discourse-based pedagogy was presented as a promising practice (e.g., Felton, Anhalt, & Cortez, 2015; Hull, Balka, & Miles, 2013). Having established a research-based background for discourse as an instructional practice for engagement and cognitive rigor, the final sections makes the connection back to middle school mathematics. *Enacting Discourse as a Mathematics Pedagogy* articulates findings to support the teacher's role in supporting discourse (e.g., Herheim, 2015; Kazak, Wegerif, & Fujita, 2015) and *Discourse and Standards for Mathematical Practices* links teacher actions to mathematics learning (e.g., Boaler, 2013; O'Connell & SanGiovanni, 2013).

### **The Literature Review Process**

The literature review process began with an analysis of resources and research to support the use of discourse in content areas other than mathematics. This phase of the review sought to understand what evidence existed to support the use of discourse as an instructional strategy. The results of this search helped to create the early sections of the literature review, specifically relating to theories that support the use of discourse and the connection between discourse and socio-cultural learning processes.

The next phase of the research process relied heavily upon the online database search function through the library at Concordia University. As additional evidence was sought to connect discourse to mathematics, information from the initial review of other content areas

(researchers, research publications, theoretical frameworks, etc.) was entered into the search along with search terms related to middle school and mathematics. The reference section of each article and study was examined in an attempt to identify key researchers and authors in the fields as well as journals and publications that seemed to yield useful resources on the topic. Additional searches were conducted to further excavate articles and studies from key authors and within key journals focused on mathematics education.

The final phase of the literature review process was intended to better understand information written for middle school mathematics educators. Particular attention was given to resources published within the past five years, since the adoption of Common Core State Standards. Resources intended for an audience of teachers often included suggestions for instructional practice and this information is presented in the final section of the literature review along with research studies that support or refute the practices suggested.

### **Background to the Problem**

Waning student engagement and achievement in mathematics during the middle school years has been a well-documented phenomenon. Scores on 2015 National Assessment for Educational Progress (NAEP) showed that 33% of eighth grade students were proficient or above in mathematics – compared to 40% proficient or above in fourth grade (Nation's Report Card, 2015). While grade eight NAEP scores rose from 15% in 1990 to 36% in 2013, the grade four scores reflected greater improvement in the same time frame and average scores for 17-year-olds were stagnant since 1973 (NCTM, 2014). A number of studies were conceived to better understand the link between declining scores and decreased engagement in middle school mathematics (e.g., Boaler, 2013; Martin, Anderson, Bobis, Way, & Vellar, 2012; Martin, Way,

Bobis, & Anderson, 2015; Thompson, Kersaint, Richards, Hunsader, & Rubenstein, 2008).

Results suggested a positive correlation between mathematics achievement and engagement.

Declining mathematics engagement and achievement in the middle school years is not a phenomenon unique to the United States. The 'middle-years dip' has also been widely reported and researched in Australia and New Zealand where studies focused on both engagement and achievement, and often both (e.g., Attard, 2013; Ayotola & Adedjei, 2009; Bobis, Anderson, Martin, & Way, 2011; Darragh, 2013). In a review of international research on mathematics anxiety and attitudes toward math, Hannula (2012) noted that, although differences between countries exist, there is an “overall tendency for students' relations with mathematics to become more negative over the school years” (p. 138). Lewis (2013) noted that data from student surveys as part of the Trends in International Mathematics and Science Studies (TIMSS) confirmed not only that mathematics achievement was low, but also that attitudes and confidence declined from grade four to grade eight. Way, Reece, Bobis, Anderson, and Martin (2015) confirmed that dual issues of under-participation and under-achievement were well documented in research, but they also contended “the 'middle-years dip' in mathematics is not inevitable” (p. 628).

Although Hannula (2012) asserted that the causal direction of the relationship was from attitude to achievement, Way et al. (2015) stated that this relationship (which they refer to as engagement and achievement), although positively correlated, was not necessarily causal in either direction, as least not in the short-term. The Organization for Economic Cooperation and Development (OECD, 2014), in its review of assessment results and data collected from student questionnaires, stated that they were not able to determine cause and effect, but the OECD suggested the need to consider not only education outcomes but also non-cognitive aspects which influence outcomes, such as students' attitudes towards learning. The impact of these dual

issues is sufficient to warrant further investigation into potential remedies and interventions. As a result of the combined decline experienced through middle school, students overall have taken less challenging mathematics courses throughout high school and their preparation for college courses has been insufficient.

In an increasingly global economy, math skills are considered essential. In its review of 2012 Programme for International Student Assessment (PISA) results, the OECD (2014) asserted “proficiency in mathematics is a strong predictor of positive outcomes for young adults, influencing their ability to participate in post-secondary education and their expected future earnings” (p. 6). The OECD’s Survey of Adult Skills found that foundation skills in mathematics have a major impact on individuals’ life chances. The survey results showed that poor mathematics skills severely limit people’s access to better-paying and more-rewarding jobs; at the aggregate level, inequality in the distribution of mathematics skills across populations was closely related to how wealth was shared within nations. Beyond that, the survey results suggested that people with strong skills in mathematics were also more likely to volunteer, see themselves as 'actors in' rather than as 'objects of' political processes, and were even more likely to trust others. Fairness, integrity, and inclusiveness in public policy thus also hinged on the mathematics skills of citizens. (OECD, 2014, p. 6). In OECD countries, more than one in five 15-year-olds failed to obtain a score of at least 2 (the baseline level of performance), thus limiting their potential to pursue mathematics courses beyond compulsory coursework.

The National Council of Teachers of Mathematics (NCTM, 2014) indicated that only about 44% of high school graduates in the United States in 2013 were considered ready for college work in mathematics, as measured by ACT and SAT scores. Still fewer, only 16% of 2013 graduates, were both proficient in mathematics and interested in a STEM career (U.S. Dept.

of Ed., 2014). In its review of 2012 PISA data, NCTM (2014) pointed to declining scores from 2003 to 2012 and the United States' placement as 26<sup>th</sup> out of 34 in its cohort of 15-year-olds' "capacity to formulate, employ, and interpret mathematics in a variety of real-world contexts as a call to action" (p. 2) to justify a call to action. This data suggested that there was indeed a problem with inadequate mathematics achievement in the United States. A lack of ability related to mathematics, often correlated with a lack of interest or engagement with mathematics, seemed to be shaped in middle school but its results could affect the remainder of one's life.

### **Lessons Learned in Other Grades and Content Areas**

In Maine, there has been a significant presence of literacy coaches within schools and districts. Instructional practices advocated by literacy coaches have provided an avenue for educators to reflect on and adapt instructional practices related to reading and writing. In recent years, teaching and learning of science has also been supported through several regional initiatives targeting implementation of Next Generation Science Standards (NGSS). Lessons learned from these content-specific initiatives have provided a unique opportunity to reconsider teaching and learning practices related to mathematics.

Professional development related to new standards for both English Language Arts and Science and the suggested instructional strategies indicated a consistent shift away from pure content knowledge toward the *use* of content knowledge for conceptual understanding and decision-making. There appeared to be a growing recognition that content knowledge alone had little use until it was applied in the formation of new knowledge and decision-making. This was a significant change and it represented a multi-faceted shift in the way educators thought about teaching and learning. No longer could education simply rely on knowing facts and memorizing procedural steps.



The changes experienced in education were partly the result of efforts to recognize and improve the ways in which education prepared students for their futures. Conley (2014) acknowledged the challenges of deeper learning at the classroom level in which he identified teacher understanding of subject matter and instructional techniques used to facilitate deeper learning as the two most significant hurdles in realizing the change needed to prepare students to be college and career ready. Indeed, the Common Core State Standards for English Language Arts (2010) describe in the introduction the intended integration of content knowledge with active application of the skills and behaviors associated with literacy for the twenty-first century. The standards and the instruction needed to meet the expectations described within the standards will require multiple experiences with critical reading, opportunities to practice responsible citizenship, and exposure to broad worldviews as students engage with high quality literary and informational texts.

Likewise, the Next Generation Science Standards (2013) outline expectations for students at each grade level as they apply content knowledge to make sense of deeper concepts and apply knowledge to design solutions to real-world problems. Students at all grade levels are expected to *use* their knowledge to engage with science at a deeper cognitive level. Next Generation Science Standards (NGSS) clarify that Kindergarten students should be able to integrate their knowledge of push and pull as they “demonstrate grade-appropriate proficiency in asking questions, developing and using models, planning and carrying out investigations, analyzing and interpreting data, designing solutions, engaging in argument from evidence, and obtaining, evaluating, and communicating information” (NGSS, 2013, p. 4). These skills and proficiencies reflect the types of deeper learning and ambitious standards Stewart (2012) suggested would be required for future graduates and citizens of a global society.

Professional development activities designed to acquaint educators with the expectations of the new standards illuminate the notion that it is not only the content of the standards that differ from past standards, but the mental processes involved in meeting these expectations are significantly more complex than in the past. In addition to general professional development activities designed to help Maine teachers address the new science standards, two particular activities stood out as significant. First, during the 2013–2014 school year, regional science work with teachers focused on the use of classroom talk to engage students in sense-making around scientific concepts. Teachers participated in a series of workshops and training sessions to create classroom norms for conversation and to promote evidenced-based interactions between students based on suggestions from Michaels, Shouse, and Schweingruber (2007). The response was phenomenal, from both the teachers and the students. Rather than being directive, teachers engaged students in observing, wondering about, and investigating science concepts. Where the expectations of the NGSS had originally seemed daunting and unrealistic, it had become clear that students could engage with science content on a conceptual level.

To support literacy instruction, during the 2014–2015 school year the Maine Department of Education offered a series of webinars and regional meetings to further develop content area conversations; the discussions were centered around two texts related to classroom discourse (Fisher, Frey, & Rothenberg, 2008; Nichols, 2006). Nichols (2006) presented a common sense approach for engaging students in deep conversations about texts and text-based evidence. She suggested that teachers ask students to reflect on *Why* an author includes certain information and to analyze the impact of those choices. Participants walked away from the training sessions with an awareness of the potential for classroom discourse to serve as a means for supporting students

as they construct new ideas, cultivate metacognitive abilities, create communities around ideas, and focus on process and strategy (Fisher, Frey, & Rothenberg, 2008; Nichols, 2006).

Professional development provided in Maine over the past three years related to the Common Core State Standards for English Language Arts as well as the Next Generation Science Standards has tended to focus less on content and more on cognitive processes involved with learning about content. Student discourse has been emphasized as a means for achieving deeper cognitive engagement. The potential of student discourse to affect mathematics teaching and learning is a question worthy of deeper consideration.

### **Conceptual Framework**

The way that discourse is implemented in any given classroom is highly dependent upon many different factors working together in a complex system. While professional development may be provided to assist teachers with establishing norms for productive discourse, the way that each teacher implemented and reinforced such norms depended on the individual teacher and the interplay of people and contexts within the classroom. Furthermore, in a specified research environment in which discourse is the known focus, a teacher's emphasis may vary following the study. Real change must be motivated from within. As such, this research project attempted to engage the teacher in reflective practice with video recording, transcript analysis, and reflective interviews using an Enactivist approach.

Enactivism recognizes the centrality of the researcher to the research process and seeks to offer an alternative to the limitations realized by the impact of the researcher's emphasis of a particular theoretical perspective (Reid, 1996). Instead, by observing teachers and students in the everyday practice of teaching and learning, participants in the research process (in this case the

teacher(s) and the researcher together) seek to use the data to uncover new understandings based on their own interest about the data. Reid (1996) stated,

Enactivist research differs from collaborative research in that there is no common goal or question in which we are all interested ... [while each user of the data is pursuing his/her own interest,] ... we work with a common collection of data, about which we each reach conclusions related to our own interests and theories. (p. 5)

In this way, an enactivist approach results in the co-emergence of theories and allows the classroom teachers to make his/her own interpretations. As the agent of change, the teacher could choose to make modification based on his/her analysis of the data, essentially engaging the classroom teacher in a form of action research.

The enactivist approach has offered a new perspective on teacher education. Brown and Coles (2012) explained the value of teachers being able to reflect on lessons to see possibilities within a classroom setting. Each person's background, interests, and experiences draws their attention to different aspects of a single lesson when viewed together; “perception is not the passive receipt of information, but an active process of categorization made possible by our history of interaction” (Brown & Coles, 2012, p. 221). Brown and Coles believed that through an enactivist approach (such as video review), teachers would observe patterns of interactions over time that bring awareness of processes and practices that impact student learning. Mathematics research projects conducted using enactivist approaches highlight the variety of topics that emerge inductively through analysis of classroom data (Coles & Scott, 2015; Reid, 2014). Davis (1999) explained the potential of enactivism in mathematics education as allowing teachers to abandon 'prescribed' methods, thus 'proscribing' ineffective practices in favor of reflective teaching within the dynamic and complex spaces of the classroom.

An enactivist approach allows for the flexible integration of psychological (individual) and socio-cultural (social/group) perspectives within the research process, recognizing that teaching and learning involve complex, imperfect, and inter-related processes. Within this flexibility, teacher metacognition and responsive action are allowed to co-emerge naturally (Davis, 1999). Because of its reliance on hermeneutic cycles, researchers using an enactivist approach must be willing to tolerate ambiguity, persevere through uncertainty, confront dissonance, and demonstrate openness to the possibilities identified within the data.

This study was developed to investigate the ways that dialogue was used as a pedagogical strategy to engage middle school students with mathematics. The intent was to openly engage the teacher in an authentic process of “deliberate analysis” (Brown & Coles, 2012) that “aims to provoke new distinctions or new awarenesses ... rather than trying to establish fixed results or truths” (Coles & Scott, 2015, p. 133). From a perspective of simultaneously improving student engagement with standards for mathematical practice *and* supporting teachers’ professional practice, the enactivist methodology allowed the researcher to sit with the teacher in an open, non-threatening format to consider student learning and the impact of teaching practices. While the main interest was classroom discourse, the teacher in this study may have chosen to notice other issues such as patterns related to group work, the use of concrete learning manipulatives, or strategies for formative assessment. As Reid (1996) pointed out, a common set of data was examined but each participant filtered it through his own interests and perspective.

## **Review of Research Literature**

**Emphasis on discourse as mathematics pedagogy.** In August 2008, the National Council of Teacher of Mathematics (NCTM) convened a group of mathematics researchers and practitioners to identify research topics that were presumed to have a significant impact on the

teaching and learning of mathematics. The resulting report identified a proposed research agenda intended to meet the top ten needs of mathematics practitioners. Classroom discourse, while not specifically addressed in the ten research priorities, was identified as a characteristic of instructional practice that may be useful to students perceived as having difficulty with mathematics. The report states, “The field would better understand how effective teachers plan for and implement the kinds of classroom discourse patterns that help important mathematical ideas surface for discussion” if research could address the question of interventions for struggling students (NCTM, 2008, p. 27). It was thus implied that classroom discourse is an instructional strategy that should be a component of improved mathematics achievement.

Since 2008, NCTM's *Mathematics Teaching in the Middle School* publication featured multiple articles that suggested to mathematics educators that classroom discourse was an appropriate strategy to help students attain greater conceptual awareness of mathematical content. Middle school teachers such as Edwards (Edwards & Townsend, 2012) conducted action research in their classrooms upon which they have reflected and concluded, “The lack of engagement was evident from the amount of unproductive talk in the classroom” (p. 175). Within two years of changing his practice, Edwards stated, “I daily encounter evidence that my students were developing deeper understandings, having richer conversations, and enjoying the learning of important mathematical topics” (Edwards & Townsend, 2012, p. 178). Initially, Edwards' focus was on instructional changes he had made such as integrating more hands-on activities, making better use of technology, and diversifying his assessment techniques. His analysis of these changes, however, focused on the quantity and quality of student discourse within the classroom as indicators through which he was able to judge students' content-based engagement and achievement.

Similarly, a number of articles were included in the NCTM publication that drew attention to the importance of the Common Core Standards for Mathematical Practice (SMP). These articles, many of which were written by middle school mathematics teachers, suggested ways to modify instructional practices to include SMPs through increased student discourse. Establishing a classroom environment that supports student interaction was identified as an essential step in integrating SMP (e.g., Stephan, 2014; Suh & Seshaiyer, 2013; Wilburne, Wildmann, Morret, & Stipanovic, 2014). As norms for student interactions were taught and reinforced in the middle school mathematics classroom, students were encouraged to 'borrow' ideas and strategies from their peers to increase their own cache of mathematical approaches (e.g., Buchheister, Jackson, & Taylor, 2015; Hull, Balka, & Miles, 2013; McGinn, Lange, & Booth, 2015; Stephan, 2014). The process of sharing and borrowing ideas and strategies to engage with mathematical concepts provided opportunities for students to discriminate between helpful and unhelpful information. Thomas, Fisher, Jong, Schack, Krause, and Kasten (2015) explained, while students listened to others and wrestled with ideas, “they are negotiating situations that provoke disequilibrium” (p. 241). The need for norms for productive discourse was evident in such cases, as some students may not have been comfortable dealing with conflicting information and cognitive dissonance. Specific SMPs also set out expectations for students to confront cognitive tension as they *make sense of problems and persevere in solving them* (SMP 1) and *construct viable arguments and critique the reasoning of others* (SMP 3).

If educators are to provide students with opportunities to be successful with middle school mathematics content and the SMPs, classroom practices are needed to confront both content and process. Through its peer-reviewed journal for middle school mathematics teachers, NCTM sought to provide a resource for educators to address this need. Based on information

provided within many of the articles since Common Core's inception, discourse was presented as a reasonable vehicle for providing effective mathematics instruction in the middle school classroom. Classroom teachers or faculty members working with pre-service and in-service teachers generally contributed these articles. As such, they drew from a great deal of personal experience and both formal and informal action research. While many of the articles suggested common themes for instructional emphasis, findings resulting from more specific studies and research projects helped to deconstruct the factors of effective classroom discourse.

**Theories supporting the use of discourse.** Discourse in education has long been considered a critical characteristic of teaching and learning. Socrates emphasized the role of the teacher to guide student discovery through discussion and questioning (Fisher, 2013; Nystrand, 1997; Reed, 2010). The Socratic method was based upon a belief that each person had background and experience that helped shape new understanding; a knowledgeable and capable teacher can help guide the process of meaning making within a 'community of inquiry' (Fisher, 2013; Reznitskaya, Glina, Carolan, Michaud, Rogers, & Sequeira, 2012). Likewise, Plato envisioned dialogue as a process for learning through which participants developed deeper understanding and creativity (Plato, 2006). Although Dewey's primary focus was on the critical role of student interest to guide and sustain educational pursuits, Dewey saw classroom discourse as a resource for promoting student interest (Hodge, Visnovska, Zhao, & Cobb, 2007). While historical perspectives of discourse for learning may have varied slightly, the role of conversation to support new learning has been universally considered a valuable pedagogy.

Vygotsky, Piaget, and Bakhtin each added to the focus on classroom discourse as a means for both student engagement and more specifically for meaning making. Vygotsky and Piaget each noted the potential for discourse to develop personal meaning within the individual.



More precisely, Vygotsky conceived of discourse as a *tool* to be used in the process of meaning making (Kazak, Wegerif, & Fujita, 2015; Reznitskaya et al., 2012). In his conception of the zone of proximal development, Vygotsky reasoned that discourse with a 'more knowledgeable other' was necessary to mediate the learning process (Mason, Drury, & Bills, 2007). Similarly, Piaget argued that discourse was a causal process whereby participants confronted disequilibrium (such as that introduced along the zone of proximal development) introduced through dialogic relationships (Kazak, Wegerif, & Fujita, 2015).

Contemporary analysis of discourse attempted to distinguish the variations between Vygotsky and Piaget's perspective of discourse as an internal, psychological process, as opposed to a more external, social approach such as that described by Bakhtin. From Bakhtin's perspective, new learning occurred through nuances and differences that emerged in conversations. Similar to Vygotsky's description of the zone of proximal development, Bakhtin believed that meaning-making required a degree of cognitive tension; “if two voices in dialogue were to coincide with each other then the flow of meaning would cease” (Kazak, Wegerif, & Fujita, 2015, p. 107). While Bakhtin viewed learning as a result of an external process of negotiation and meaning making contingent upon conversations (and incompatibility), there was a commonality expressed within each of the theories that discourse between two or more people was important for the genesis of understanding.

Researchers seeking to analyze the role of discourse in education have consistently referred to the emphasis that dialogic theories place on the process of learning how to learn (e.g., Herheim, 2015; Mercer & Sams, 2006; Monaghan, 2006; Wegerif, 2008). Discourse provides an opportunity for students to verify their understanding, to seek clarification of new information, and to apply prior knowledge to new situations. Questioning, confronting uncertainty, and

engaging with cognitive complexity have been identified as essential features of discourse-based learning environments (Reznitskaya, et al., 2012). A yearlong study of teacher practices suggested that instructional practices be considered in light of their potential for supporting students as they safely engage with new learning, while preserving and protecting their identities (Turner, Warson, & Christensen, 2011). Such studies confirmed that instructional practices have the potential to mediate positive and negative motivational forces affecting student learning.

**Engagement in the learning process.** Based on qualitative analysis of classroom transcripts, Reznitskaya, Glina, Carolan, Michaud, Rogers, and Sequeira (2012) acknowledged, “learning in a dialogic classroom is predicated on active engagement” (p. 303), suggesting that students must actively participate in discourse opportunities if they are to benefit from the learning opportunities provided. While discourse was shown to be a common feature identified in case studies of highly engaged mathematics classrooms (e.g., Attard, 2013; Brooks & Dixon, 2013; Way, Reece, Bobis, Anderson, & Martin, 2015), engagement itself is a multi-faceted construct. Longitudinal, mixed methods studies attempted to identify factors involved in middle school engagement and the mediating effects of teacher supports (Attard, 2013; Skinner, Furrer, Marchand, & Kindermann, 2008).

Engagement has been viewed in relation to affective (or emotional), behavioral (operative), and cognitive components (Attard, 2012; Bobis et al., 2011; Hannula, 2012; Skinner et al., 2008). In a four-year longitudinal study of student engagement conducted in upstate New York, Skinner, et al. (2008) applied quantitative analyses to data collected through questionnaires in order to create a construct for engagement and disaffection in middle school classrooms. Results of the study supported findings from previous studies and identified both student-centered behaviors such as social withdrawal and lack of participation (Hannula, 2002)

and teacher-centered behaviors such as failure to implement active and engaging pedagogies (Balfanz, Herzog, & Mac Iver, 2007) as problematic.

Hannula (2002) specifically addressed student attitudes as the combined embodiment of cognitive, affective, and behavioral engagement. Hannula explained that emotions were consequences of cognitive processes related to personal goals, and behaviors were directed by emotions. Thus, the progression was viewed as moving from cognition to emotion to behavior. As students worked toward their goals, they perceived situations and information as either helping or as hindering goal achievement. Helpful situations were associated with positive emotions and adaptive behaviors. Situations perceived as hindering goal achievement were associated with negative emotions such as fear or sadness and subsequently either active or passive avoidance behaviors.

In 2012, Hannula revisited several theories about student engagement in an attempt to construct a framework for math-related affect. Reiterating the findings of Green, Martin, and Marsh (2007) and Sullivan and McDonough (2007), Hannula analyzed the connection between motivation and student perception as a mediator of engagement. He explained that students who perceive mathematics content as useful to their future were more likely to have a positive affect and therefore more willing to engage in activities related to learning activities. In contrast, students who did not readily see the relevance of course work to the attainment of their goals were less likely to approach the work with a positive attitude. Hannula (2012) suggested that attention be given to the psychological factors influencing student engagement in class activities and that mathematics instruction seek to accommodate psychological needs. Willingness to accommodate student goal orientations and relevance were directly linked to likelihood of giving up if a student encountered difficulty or in avoiding challenging activities altogether. Recent

research on student engagement offered a caution, however, as student goals that valued outcomes (such as getting an A) over *learning* led to fear of failure and had the potential to significantly hinder students' willingness to engage in challenging learning activities (e. g., Boaler, 2013; Dweck, 2006; Way et al., 2015).

***Socio-cultural learning and engagement through discourse.*** As educators consider students' willingness to engage with content, consideration for personal goals should not be ignored. Socio-cultural learning theory has suggested that students' social goals are highly motivating (Mercer & Sams, 2006; Sullivan, Mousley, & Zevenbergen, 2006). Indeed, Attard (2012) claimed, “the social element of learning is critical to students in the middle years” (p. 11). Social learning experiences as a means for academic learning and identity development have been linked to and intertwined with the dialogic learning processes described by Vygotsky, Piaget, and Bakhtin. Researchers and educators who understand this connection acknowledge that mathematics learning “happens through participation in a social ecology and through the processes of identity development and communication” (Darragh, 2013, p. 216). As such, efforts to engage students with mathematics should take into consideration individual identity goals, which are closely tied to social processes. As students engage in discourse around content-based topics, they have demonstrated the ability to adapt and refine their mathematical identities as well as their social identities.

As educators seek to promote content-specific goals, they cannot overlook students' own goals and priorities. To realize the potential of student engagement, educators should seek to find ways to attend to content-based goals while honoring students' individual goals. Instructional practices that encourage students to work together to make meaning of content through socially negotiated “interactions with others, and through the historical and cultural norms that operate

within their lives” (Attard, 2013, p. 573) have demonstrated potential to meet both personal and content-based goals. Such practices have included attention to an interactive learning environment and supportive class culture (Brooks & Dixon, 2013; Walshaw & Anthony, 2008); opportunities for active, hands-on exploration focused on mathematical concepts (Attard, 2012; Cheeseman, 2009; Way et al., 2015); and appropriately challenging activities that allow for multiple access points or approach strategies (Billings, Coffey, Golden, & Wells, 2013; Coles & Scott, 2015; Sharma, 2015).

A positive learning environment that honors the safety of all participants has been identified as a fundamental requirement for students to engage in social learning. Sharma (2015) asserted, “learning environment and classroom culture are major contributors to success for students” (p. 300). Research suggested that teachers more effectively engage students in learning when they were able to create spaces for and instruct students in ways to think creatively (Stephan, 2014). Creating learning activities that introduced ways to confront and embrace ambiguity promoted engagement (Barwell, 2005; Brown & Coles, 2012) and provided opportunities for students to accept and make sense of multiple perspectives. Engagement flourished where students were able to encounter strategies and ideas that differed from their own (McGinn, Lange, & Booth, 2015) and the teacher was able to support students as they practiced strategies to persevere in challenging situations (Felton, Anhalt, & Cortez, 2015; Wilburne et al., 2014). Minimizing perceived student risks associated with active engagement means that educators must attend to psychological and social goals of students within the mathematics classroom and curriculum. Effective pedagogical practices enacted by a noticing teacher have great potential for contributing to a quality-learning environment and mathematical outcomes.

**Discourse for higher order thinking and cognitive rigor.** The Common Core State Standards and their accompanying assessments articulate high expectations for student knowledge and the processes they should engage in as they apply content knowledge. Whether educators refer to Bloom's revised taxonomy of cognitive processes (Anderson & Krathwohl, 2001; Pohl, 2000), Webb's depths of knowledge (Webb, 2002), Marzano's dimensions of learning (Marzano & Kendall, 2008), or Hess' cognitive rigor matrix (Blackburn, 2014), instruction and assessment have increasingly emphasized greater cognitive demand and the generative possibilities that exist through the application of knowledge and skills. More than simply memorizing rules, applying mnemonic devices, or following steps in a process, students have been expected to look for and use patterns, structures, and evidence of repeated reasoning to make sense of new content and to critique processes used by others. Hull, Balka, and Miles (2013) asserted, “to meet this shift in assessments and demand for rigor, classroom instruction must change” (p. 52). To meet the expectations of the new learning standards, instructional practices will need to move students from passive recipients of information to active participants in the learning and meaning-making process (Hand, 2012).

The shift in focus from mathematics as a having a single, objective response to the idea that mathematics concepts present multiple avenues for considering subjective problems has been seen as a challenge to traditional instructional practices. While the Standards for Mathematical Practices (SMP) demand attention to precision (SMP #6), there has also been increasing emphasis on ambiguity as a *resource* for teaching and learning mathematics (Barwell, 2005). Here, it was important to draw the distinction between scaffolding and rescuing. Scaffolding refers to the teacher supports and prompting offered to students as they encounter new learning. Scaffolds may be presented in the form of increasingly difficult or abstract

problems, questions that prompt students to think differently about a problem, or suggestions for new strategies to try (Alibali, 2006; Larkin, 2002). The goal of scaffolded instruction is to help students identify and apply what they already know and to generalize previous knowledge and skill to new learning situations. While scaffolding learning activities to promote success has been viewed as an important feature of effective pedagogy, if not monitored closely, scaffolding can become rescuing. Rescuing occurs when the teacher provides so much support that students no longer need to apply their knowledge or make decisions about which procedures would be most effective. By rescuing students from the process, educators risk removing the cognitive challenge and sending the message that the students are incapable of completing the work on their own (Thompson, 2010; Walls, 2007). Learning appeared to be minimized or completely removed when educators rescued, rather than provide scaffolding.

In consideration of Vygotsky's zone of proximal development, scaffolds in the form of instructional activities and prompts are viewed as important to guide students to the edge of new learning and discovery, but new learning occurs when students are able to internalize the information and reconcile incompatibilities. As faculty members responsible for training and supporting mathematics educators, Felton, Anhalt, and Cortez (2015) stressed the role of the teacher in terms of the questions and verbal prompts used to challenge students' use of mathematical models. Fenton et al. emphasized, "the teacher is of particular importance in helping students understand the context, questioning their assumptions, and considering whether a model is adequate or should be revised" (p. 348). Likewise, Thomas et al. (2015), also faculty members in mathematics education, explained that discourse-rich mathematics classrooms provided a unique opportunity for students to "wrestle with ideas and listen to others while they

are negotiating situations that provoke disequilibrium” (p. 241) under the watchful eye and guidance of a teacher who was able to skillfully guide students along the learning trajectory.

***Demand for cognitive rigor and the promise of challenging standards.*** While the development of high expectations and rigorous, internationally-benchmarked standards have been recognized as a step in the direction toward improved student achievement and a world-class education, changes in instructional practice are needed if the potential of the new standards are to be realized (Stewart, 2012; Conley, 2014). Darling-Hammond (2010) argued that high-quality instruction was essential for promoting educational equity for American school children. Darling-Hammond noted the findings of a National Education Longitudinal Study in which cooperative learning was identified as a common practice among schools with significant and more equitably distributed achievement gains. “Intellectually challenging and relevant instruction” (p. 250) characterized by “careful scaffolding for the learning of complex skills” (p. 252), was outlined by Darling-Hammond as necessary for promoting high achievement. Such emphasis was supported by Hull, Balka, and Miles' (2013) call for greater cognitive rigor demands and by Felton, Anhalt, and Cortez' (2015) explication of the role of the teacher in supporting deeper understanding by students. Likewise, following a systematic review of education systems around the world, Hargreaves and Shirley (2012) determined that high-achieving systems consistently “support learning in depth rather than superficial coverage of curriculum content” (p. 176). Hargreaves and Shirley referred to such practice as 'teaching less to learn more,' terminology that underscored the focus on learning as the intended outcome of instructional practices. It seemed clear that the role of the teacher and the strategies used by the teacher during instructional activities were consistently linked to greater student engagement and more equitable achievement



**Enacting discourse as a mathematical pedagogy.** In real-world situations, students will be required to apply their knowledge to make judgments and determine the reasonableness of mathematical situations. Providing students with challenging learning opportunities that require them to engage in higher order thinking processes are important, but instructional practices are needed to support students in taking the risk to engage in challenge (Sharma, 2015). Sharma noted, “ample class time should be spent on discussion and reflection rather than presentation of information” (p. 300). Echoing the practices of high-achieving schools identified by Hargreaves and Shirley (2012), Coles and Scott (2015) referred to the practice of 'subordination of teaching to learning' and highlighted the importance of teachers attending to evidence of student learning and subsequently implementing responsive instruction based on student needs. Classroom discourse provided an opportunity for teacher to elicit evidence of student learning along a trajectory of learning within the content knowledge (Thomas, Fisher, Jong, Schack, Krause, & Kasten, 2015) and to plan future instruction for movement along the trajectory.

Much attention has been given to the strategies teachers have utilized for creating classrooms where discourse was expected as part of mathematics learning. Stephan (2014) outlined a process for establishing social norms that promoted reasoning, questioning, and agreeing or disagreeing. Likewise, Herheim (2015) noted the need to “create a space where ideas can be expressed and discussed” (p. 109). Based on the results of a large-scale, mixed method, experimental program study, Monaghan (2006) pointed out the need for teachers to be actively involved in a continuous process of modeling, monitoring, and reinforcing, during student discussions and group work. While building mathematical content knowledge and facility with higher order thinking around mathematical concepts was a primary goal, students needed to be supported in this goal through effective teacher pedagogies. Monaghan suggested that student

discourse be viewed as a means of 'thinking aloud with others.' Emphasis on the central role of thinking during classroom discussions aligned with Bakhtin's conception of discourse as a process for meaning making through interactions with others (Kazak, Wegerif, & Fujita, 2015).

Discourse as a pedagogy supports the socio-cultural perspective of learning in which safe learning environments arise through relationships among students and between students and teachers. Studies seeking to identify characteristics of effective student-teacher relationships provide guidance to teachers who seek to support students' mathematical identities. George (2009), emphasized the significance of relationships to mathematical identity and acknowledged, “many students form a relationship with mathematics mediated by the relationship they have formed with their mathematics teacher” (p. 211). Lerman (2009) and Davis and Williams (2009) each noted the use of discourse as a means for teachers to monitor students' multiple identities in the mathematics classroom. Specifically, discussion-based instruction supported student identities by “allowing for a range of personalities, learning preferences, modes of expression, and work rates” (Bobis et al., 2011, p. 35), which simultaneously promoted engagement and motivation in middle school mathematics courses.

A longitudinal study of 20 middle school students over a three-year period focused on students' perceptions of factors influencing their engagement with mathematics (Attard, 2013). Patterns were identified from interviews, group discussions, and classroom observations that revealed key factors and their resulting impact on student-reported engagement. Results of the study suggested that students were more engaged when their teacher used a greater variety of instructional techniques and when they “felt their teacher 'knew' them in terms of their learning needs” (p. 582). Way et al. (2015) sought to expand upon the research of Attard (2013) to better understand the specific interactions between students and a teacher that supported motivation and

engagement in middle school mathematics. A mixed-methods phenomenological study conducted by Way et al. looked at data from a grade six classroom where engagement and motivation levels were higher than expected. Inductive analyses of classroom observations, field notes, and teacher interviews related to this study indicated common themes within the data. Overall, interactions between students and the teacher were characterized by frequent probes by the teacher to monitor student progress, teacher prompts to support (or scaffold) student engagement, and student and teacher conversations focused on mathematical thinking (Way et al, 2015).

Attard (2013) and Way et al. (2015) explored how effective interactions between students and teachers were characterized by teacher knowledge of learning progress and strategic use of learning activities to move students along the progression. Teachers used discourse as a means of determining current levels of achievement, prompting students to extend their knowledge to progressively more complex situations, and providing feedback to students. As such, the use of discourse as an instructional pedagogy has been shown to have the potential to subordinate teaching to learning (Coles & Scott, 2015) as educators focus on eliciting evidence of mathematical practices.

**Discourse and standards for mathematical practice.** The Standards for Mathematical Practice (SMP) are outlined in the introduction to the Common Core State Standards for Mathematics (National Governor's Association & Council of Chief State School Officers [NGA & CCSSO], 2010). Eight SMPs (Appendix A) were developed from a combination of the process standards of the National Council for Teachers of Mathematics (2000) and the strands for mathematical proficiency identified by the National Research Council (2001). The SMPs are intended to be equally weighted with the grade level Mathematics Content Standards identified

in the Common Core. In addition to the detailed description of the SMPs included in the introduction to the mathematics standards document, SMPs are also reiterated on the first page of each set of grade level standards. The inclusion at the outset of each grade level serves as a reminder to educators that instructional practices should seek to develop the skills, processes, and dispositions students will need to engage deeply with mathematical content.

Content standards that define expectations for *understanding* are meant to be viewed as particularly important as these standards identify critical 'points of intersection' between content standards and SMPs. The authors of the Common Core mathematics standards explained, “these points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve ... student achievement in mathematics” (NGA & CCSSO, 2010, p. 8). As students engage with content such as fractions, they should use the mathematical practices to deepen their understanding. Prior knowledge about the structure of fractions (SMP #7) as well as diagrams and representative models (SMP #4), should be used to help students communicate their understanding and critique the reasoning and strategies of others (SMP #3). O'Connell and SanGiovanni (2013) explained, “mathematically proficient students are able to do more than provide an answer; they are able to justify that answer and defend their process for finding the answer” (pp. 43–44), thus linking mathematical content with standards for mathematical practice.

The SMPs are as much about what the students are able to do as they are about the opportunities teachers create through pedagogical decisions. NCTM (2014) suggested that educators implement discourse as a resource to foster higher order thinking and productive struggle associated with student engagement and deep understanding of mathematical content.

O'Connell and SanGiovanni (2013) specifically linked desired student skills and productive dispositions to teacher actions for each of the eight SMPs.

**Our students are better able to ...** listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the argument, **because as teachers we ...** create student-to-student dialogue rather than relying solely on teacher-to-student discussions. [emphasis in original] (O'Connell & SanGiovanni, 2013, p. 58)

The strategies and examples provided by O'Connell and SanGiovanni align with the instructional shifts outlined by Leinwand (2009) and Schwols and Dempsey (2013) to improve student engagement and raise mathematical achievement. Drawing from research about what works in reading instruction, Leinwand advocated for language-rich classroom routines that move “from literal to inferential to evaluative” understanding, employ “questions that do not have a single correct answer,” and “dovetail with emerging brain research findings about how higher-order questions support the development of more and stronger neural connections” (pp. 15–16). These suggested practices drew from research findings and recommendations of NCTM, but they have also been found to be congruent with findings from studies on growth mindset (e.g., Boaler, 2013; Dweck, 2006), self-efficacy (e.g., Jiang, Song, Lee, & Bong, 2014; Skaalvik, Federici, & Klassen, 2015), and motivation (e.g., Davis & Williams, 2009; Kennedy, 2009) in mathematics. Evidence suggests that changes in learning outcomes are preceded by shifts in teaching practices (NCTM, 2014; Schwols & Dempsey, 2013).

The National Council of Teachers of Mathematics (2014) expanded upon the teacher practices identified by Leinwand (2009) and O'Connell and SanGiovanni (2013). NCTM sought to identify instructional 'actions' to integrate mathematics content knowledge with standards for

mathematical practice. NCTM expounded a “belief that mathematics lessons should be centered on engaging students in solving and discussing tasks that promote reasoning and problem solving” (p. 10). Recognizing the contribution of previous research, NCTM referred to its list of 'actions' as a 'research-informed framework' for teaching and learning that leverage high impact teaching strategies for the purpose of deep learning of mathematics. Consistent with previous instructional practices identified, NCTM placed emphasis on student discourse, effective questioning, use of evidence and reasoning, and opportunities for productive struggle as key features of effective mathematical teaching practices (see Appendix B).

## **Chapter 2 Summary**

Research conducted in the past ten years established a link between classroom discourse and increased student engagement. Discourse has been identified as a feature of instructing leading to engagement in mathematics classrooms and a positive correlation was established between engagement and achievement. Research findings related to risk-taking behaviors (a form of engagement) in mathematics classrooms suggested that learning opportunities emerged from collaborative conversations involving differing points of view as students worked together in groups. Group work was shown to have a positive influence on both engagement with mathematics and development of strategic competencies such as those identified as desirable outcomes in curriculum documents. Research presented in this chapter supports the use of classroom discourse as a means for engaging students with mathematics course content and promoting productive work habits as students confront cognitive challenges. Further connections between engagement and achievement indicated that increased student engagement in mathematics coursework is likely to contribute to mathematics achievement.

The research findings presented throughout this chapter rely largely on mixed methods studies. Several of the studies were large-scale, multi-year studies in which data from both student and teacher questionnaires supplied a significant portion of the information. Additionally, several phenomenological studies provided useful information about characteristics of learning environments in which student achievement or motivation existed at higher rates than expected. These research studies consistently indicated that the role of the teacher was of utmost importance in creating and sustaining a learning environment in which students felt challenged but also supported as they worked to meet expectations.

Literature written for and by educators suggested that discourse is an effective strategy for scaffolding student instruction along a trajectory of learning. Publications for mathematics educators also indicated the importance of student engagement in discourse-based instruction as a means for engaging in the Standards for Mathematical Practice. As the SMPs are an integral part of the Common Core State Standards for Mathematics, implemented in more than 40 states, it is important for research to address instructional practices that support the SMPs. At the time of this study, there were no published studies to address this need. Considering the evidence that suggested the potential for discourse to be an effective instructional practice in middle school mathematics, this study was implemented to better understand how middle school teachers use discourse to specifically engage students with the Standards for Mathematical Practices. Chapter three outlines the methodology used to investigate this relationship.

### **Chapter 3: Methodology**

The Common Core State Standards for Mathematics (National Governor's Association & Council of Chief State School Officers [NGA & CCSSO], 2010) include eight Standards for Mathematical Practice (SMP). The SMPs are intended to be equally weighted with the grade level mathematics content standards. Their inclusion on the first page of each grade level's mathematics standards in the Common Core document serves as a reminder to educators that instructional practices should seek to develop the skills, processes, and dispositions needed to engage deeply with mathematical content. This study was based on the premise that student engagement is positively correlated with achievement and that discourse is a means of promoting engagement within the classroom (Hannula, 2012; Martin, Anderson, Bobis, Way, & Vellar, 2012; Martin, Way, Bobis, & Anderson, 2015). As such, teacher actions that promote student engagement and interaction with mathematical concepts were the central focus of this study. The purpose of this qualitative case study was to identify discourse-based structures and practices used by a middle school mathematics teacher to support student engagement with the eight Standards for Mathematical Practice (SMPs).

#### **Research Questions**

The over-arching question addressed in this study was, *how is discourse used as an instructional strategy to engage middle school students with the Standards for Mathematical Practices?* Data collection addressed the following three questions to inform the over-arching research question.

1. How were norms, routines, and classrooms expectations established and reinforced to support student discourse?
2. How were Standards for Mathematics Practice emphasized during instruction?



3. How were teacher-centered instructional strategies implemented to engage students with discourse around SMPs?

The purpose of question one was to identify expectations that had been established for the effective use of discourse moves within the mathematics classroom. Action research projects (e.g., Buchheister, Jackson, and Taylor, 2015; Stephan, 2014) highlighted the importance of establishing routines and norms to create a safe learning environment in which students are willing to engage in complex tasks. Thomas, Fisher, Jong, Schack, Krause, and Kasten (2015) explained the need for clearly established guidelines for interaction as students learn to negotiation disequilibrium, an important component in making sense and persevering (SMP1), reasoning abstractly and quantitatively (SMP2), and constructing viable arguments and critiquing the reasoning of others (SMP3). Publications written for teachers of mathematics recommended establishing routines for discourse as means to make mathematics more accessible (e.g., Leinwand, 2009; NCTM, 2014; O'Connell & SanGiovanni, 2013).

Question two reflected the centrality of the Standards for Mathematical Practice to mathematics instruction. The Common Core math standards (2010) identify the eight mathematical practices as equal in importance to the grade level content standard. Careful attention to which standards were addressed and how they were addressed was a focal consideration for this study. At the time of this study, there were no published studies that address implementation of SMPs. Data collection related to question two provided information about which SMPs were targeted during instructional activities.

Question three reflected the integral role of the teacher to the process of sustaining student engagement with mathematics. A large scale, mixed methods study of student-teacher interactions concluded that effective interactions focus on monitoring mathematical progress,

prompting for explanations, and extending student thinking (Way, Reese, Bobis, Anderson, & Martin, 2015). Such findings supported the understanding that learning happens along a progression and that a teacher's responsibility is to support students in their movement along the learning trajectory (Thomas et al., 2015). The purpose of this question was to uncover the specific actions and decisions implemented by the teacher to support student learning. Bakhtin's view that meaning is socially negotiated offered a lens through which group processes could be considered (Wegerif, 2008); while Vygotsky's focus on the internal knowledge development resulting from discourse (Kazak, Wegerif, & Fujita, 2015) offered a means for considering the choices of the teacher during interactions. The teacher's actions related to monitoring, prompting, and extending, positioned him in a cognitive apprentice role in which student development within the zone of proximal development was supported (Mason, Drury, & Bills, 2007). Data collected for question three was used to analyze the ways that the teacher used discourse to engage students with SMPs. Analysis of the data focused on the significance of these teacher actions toward achievement of the stated learning goal.

### **Purpose and Design of the Proposed Case Study**

This qualitative case study was designed to focus specifically on the actions of a middle school teacher related to discourse and student engagement with Standards for Mathematical Practice (SMPs). Although publications written for mathematics teachers stressed the importance of utilizing student discourse as a means for practicing the skills and dispositions outlined in the SMPs, there was a gap in available research describing the choices teachers make to connect discourse to SMPs. A 2013 qualitative case study in New Zealand reported factors influencing student perspectives on their mathematics learning, but the study's author stated, "relatively little literature gives much insight into how teachers might foster positive mathematics identities in

their students” (Darragh, 2013, p. 226). Likewise, in a 2013 case study in Australia, the researcher investigated factors influencing lowered student engagement in middle school mathematics and noted, “a result of this study highlights further the importance of continued research to improve the engagement of students with mathematics” (Attard, 2013, p. 586). Attard stated that further research was needed to investigate issues surrounding mathematics teachers and “their perceptions of teaching mathematics, and their impact on student engagement” (p. 586). Another study conducted in Australia in 2015 concluded, “the specific nature of [interactions between teacher and student] in mathematics classrooms remains under-researched, and little attention appears to be given to the specifics of these pedagogical relationships” (Way, Reece, Bobis, Anderson, & Martin, 2015, pp. 629–630). The authors of these studies specifically concluded that additional research was needed to investigate the role of the teacher in promoting student engagement. This study attempted to address these identified gaps by identifying discourse-based structures and practices used by a middle school mathematics teacher to support student engagement with the eight Standards for Mathematical Practice (SMPs).

Like the 2013 studies conducted in New Zealand and Australia, the proposed study utilized a qualitative case study design. Case study was deemed appropriate since the goal was to explore, describe, or understand complex social situations or interactions (Stake, 1995). Merriam (2009) noted that case studies, by definition, allow researchers to investigate questions of process and interaction “by means of direct observation in natural settings, partly by their access to subjective factors (thoughts, feelings, and desires)” (p. 46). With the focus of this study on teacher actions, the case study design allowed the researcher to investigate student engagement from the perspective of the teacher through his actions, intended purpose, and reflection upon outcomes.

The case study approach offered the benefit of analyzing a single case with sufficient detail to understand the impact of variables within the boundaries of a specific case (Yin, 2015). The proposed study analyzed the single case of one teacher who taught mathematics at grades six, seven, and eight. Data collected within the boundaries of this case focused on the actions and variables influencing the decisions of the teacher as he worked to engage students with SMPs at multiple grades and a range of abilities.

In attempting to identify the norms, routines, and expectations for student discourse established within the mathematics classroom (research question #1), this study explored conditions which supported Bakhtin's (1981) view of dialogue as a socially mediated process of meaning-making. During the 2013–14 school year, the participating teacher engaged in a yearlong training of strategies designed to elicit understanding through increased student talk. He had implemented those strategies in his mathematics classes over the past three years. The participating teacher also utilized a conceptually focused curriculum that balanced constructivist processes with direct-instruction.

The case study design was well suited for this study because of the focus on understanding features already in place within the case under observation. Yin (2015) explained that a case study should be considered when the behaviors of those being studied will not be manipulated and when factors influencing the context are under investigation. Data collection procedures did not alter or manipulate current practices, but rather were designed to capture ordinary interactions as they naturally occurred within the mathematics classroom. As such, this study was well matched to the case study design.

## **Research Population and Sampling Method**

This study utilized a single embedded case design (Yin, 2014). Data was collected from the classroom of a middle school mathematics teacher in central Maine. The subject taught multiple sessions of mathematics in grades six, seven, and eight each day, including one class of Algebra I. The single case of the one teacher with multiple classes embedded, provided an opportunity to identify possible consistencies across classes. While the teacher was the same across this case study, the embedded study of separate classes allowed the researcher to better understand the conditions influencing the teacher's decisions, especially related to teacher centered behaviors addressed by research question #3. Interviews conducted throughout the data collection phase provided an opportunity to explore choices made by the teacher in relation to the Standards for Mathematical Practice (research question #2).

This study was conducted in the classroom of a middle school mathematics teachers. The school was a small, rural middle school serving students in grades six through eight. Although there were two mathematics teachers in the school, data was only collected from one of the teachers because the other teacher was new to the district with just one year of experience teaching middle level mathematics. The teacher who participated in the study had four years of experience in the school system and, based on observations of administrators, was interested in improving his craft while helping students learn and enjoy mathematics.

In addition to being a convenient sample, the teacher and his classroom were useful research subjects in the context of middle school mathematics education in rural Maine. It is not uncommon for teachers in rural schools to work with the same group of students over multiple years. In many schools, one teacher may provide all of the mathematics instruction throughout the entire middle school experience. Edwards and Townsend (2012) used a similar case study

methodology as they considered instructional practices of a single teacher serving all students in a middle school over three consecutive years of their education. As Edwards and Townsend pointed out, the teacher's willingness to critically analyze the significance of his practice to student learning was “critical to ensuring ongoing improvement toward best practice” (p. 175). This view also aligned with the enactivist approach to research in which the teacher actively engages in making sense of data for the overall improvement of practice – a stated goal of the participating teacher.

### **Case Study Instrumentation**

In alignment with data collection techniques outlined by Stake (1995) and Merriam (1998), this study relied on observations, interviews, and document analysis. The researcher, as a visible participant in the setting, openly conducted observations. Merriam (2009) described such involvement as 'observer as participant' where the researcher's primary role is information gathering. Field notes were recorded during each observation, focusing on elements identified by Merriam (2009) as important aspects of observational data: the physical setting; participants; verbal prompts by the teachers; context of the class (time of day, day of week, other relevant events); and class activities, including notes about events which served as topics for inquiry during subsequent interviews.

During observations, lessons were recorded and transcribed for careful analysis (as described below in 'Data Analysis Procedures'). The teacher was familiar with the process as it is common practice for teachers in the district to record their lessons to aid in reflective practice. Audio recordings were transcribed by the researcher immediately after each observation. Although Yin (2015) warned of the importance of *hearing* the account firsthand over *reading* a transcript, the verbatim transcription offered an opportunity to consider specific word choice,

activities, and prompting used by the teacher to engage students with SMPs (research question #3). Further, verbatim transcription offered a degree of reliability as it was free from researcher interpretation and provided a point of access for conducting interviews with the teacher based on observations.

Interviews with the teacher were designed to clarify which SMPs were targeted (research question #2) and to elicit the rationale for discourse moves related to SMPs (see Appendix C). Although Stake (1995) contended, “a considerable proportion of all data is impressionistic” (p. 49), the interview process allowed the researcher to check impressions against the perspective of the teacher. Merriam (2009) suggested the use of interpretive interview questions to uncover explanations of observations, and experience or behavior questions to evoke the intended purpose directing behaviors, actions, and activities. Such questions, presented in an open-ended format, supported a more nuanced understanding of the complexities involved in teacher decision making within the context of SMPs. Interviews permitted simultaneous engagement in data collection and analysis as observations shaped questions and responses to interview questions shaped future observations. Merriam (1998, 2009) suggested that data collection and analysis are recursive processes, aiding inductive thematic identification and constructivist epistemologies.

In keeping with the triumvirate of data collection instruments recommended for qualitative case study research (Merriam, 1998, 2002; Stake, 1995), this study also included document analysis. Document analysis was used to confirm alignment between formally stated norms and expectations and enacted routines practiced within the classroom (research question #1). This study relied on documents that already existed as a natural product of the setting, such as course handouts, assignments, reporting documents, and curriculum resources. Here too,

information presented in documents aided the researcher in determining which aspects of instruction to focus on during observations, as well as guiding the development of questions to better understand the connections between observed behaviors and intended outcomes.

### **Data Collection**

Permission for this study was obtained from the Superintendent and school principal before the end of the 2015–16 school year. Because of the position of the researcher within the school district, the principal helped to identify which teacher would be involved in this study. The teacher was made aware of the study and the types of data collection to be utilized.

Data collection began at the start of the 2016–17 school year. Classroom observations took place starting during the first week of school. Field notes were collected during each observation. Field notes were reviewed and annotated soon after each observation. Artifacts and documents, such as handouts from classroom activities, were collected immediately along with evidence from the classroom setting, such as floor plans and grouping charts. In addition to direct observations by the researcher, an average of one class period each week was recorded and transcribed, starting during the first week of school and extending to the end of the first grading period. Transcripts were created within 24 hours of the class being recorded.

Interviews with the teacher were conducted an average of once every two weeks. These interviews were conducted in a semi-structured format. Data collected between interviews was analyzed in an ongoing, iterative process and used to shape interview questions, reflecting Merriam's (2009) assertion that data analysis occur simultaneous with data collection. Stake (1995) considered data to be largely impressionistic, which could result in subjectivity and thus become problematic. Therefore, interviews were used to clarify initial impressions formed from observations and document review, while engaging the teacher in a process of member checking.



Data collection continued throughout the first trimester (approximately 12 weeks). The process was iterative, with ongoing analysis. The iterative nature of the qualitative case study process supported the constructivist epistemology in which new insights and understandings are constantly constructed, verified, and adjusted throughout the research process. Not only was this process appropriate for qualitative case study, but it was also an appropriate process for studying mathematics instruction. Davis (1995) suggested that mathematics teaching practices should consider co-emergent processes and phenomena occurring through social interactions that lead to more subjective constructivist orientations toward mathematics.

The SMPs define important “processes and proficiencies” that mathematics educators should “seek to develop in their students” (NGO & CCSSO, 2010, p. 6), consequently leading to greater reliance on process-oriented and learner-centered instructional practices. Throughout the data collection process of this study, attention was given to understanding the instructional practices used to support the SMPs. In consideration of continual emphasis on research questions 2 and 3, data collection focused on *which SMPs* were being emphasized and the *instructional practices* used to engage students. Interview questions were implemented to illuminate connections between the observed answers to these questions and the teacher's choice related to each.

### **Variables of Interest**

The purpose of this study was to examine discourse-based instructional strategies used to engage middle school students with the eight Standards for Mathematical Practice, as such, the two main variables of interest in this study were discourse and Standards for Mathematical Practice. The eight SMPs were each monitored separately (research question #2). Variables related to discourse included the amount of *time* the teacher's voice dominated the classroom

(through whole class instruction, leading discussion and investigations, etc) and specific *prompts* used to engage students with mathematical content and SMPs (research question #3). At a basic level, prompts were analyzed to determine whether questions were posed in an open-ended or closed format. Additionally, prompts were analyzed to determine whether they sought to extend, clarify, and evaluate. Considering the content of the SMPs, teacher prompts were also analyzed to determine the extent to which they solicited student critiques of reasoning, use of models, and mathematical justifications.

In relation to the types of teacher prompts observed throughout the study, previous research suggested many possibilities that could likely emerge. Way et al. (2015) found that effective teacher prompts were characterized by questions that served to monitor progress, feedback that guided students along a progression of learning, and emphasis on mathematical thinking as opposed to a correct answer. Likewise, Billings et al. (2013) and Thomas et al. (2015) suggested that teachers constantly monitor and scaffold for success along a trajectory of learning. Such an emphasis on monitoring and supporting student progress might suggest evidence of formative assessment. Subsequent interview questions were designed to verify this intention and to understand the possible connection in more detail. Boaler (2013), Coles and Scott (2015), and Sharma (2015), each identified connections between teaching strategies that promoted mathematical challenge and risk-taking and their positive impact on student engagement, perseverance, and a growth mindset. Here too, with the connection between teaching practices that support engagement and the 'productive dispositions' outlined in the SMPs (NGO & CCSSO, 2010), the researcher anticipated evidence of such interactions to emerge. The possibility of these variables emerging along with others were monitored continuously through the ongoing analysis process.

## **Data Analysis Procedures**

In light of the focus on discourse related instructional strategies utilized by the middle school mathematics teacher to engage students with the eight SMPs, data analysis occurred throughout the study, simultaneous with data collection. Field notes and lesson transcripts were analyzed and coded for emerging themes related to teacher behaviors to engage students in discourse and specific connections to the SMPs (research questions 2 and 3). As themes and patterns emerged, ongoing data collection was used to confirm or refute the existence of such patterns. In this way, new data collection sought to verify developing trends.

Notes and transcripts were entered into a word processing program with numbered lines. Each new entry included the date and context in which the data was collected. Codes were included in the right hand margin of the transcript and indexed electronically for later retrieval and review. Codes included references to norms for participation, which research question was being addressed (Q1, Q2, Q3a, Q3b, etc.), specific SMPs being stressed (SMP1, SMP2, etc.), and emerging themes related to the types of prompts used by the teacher or teaching strategies (individual, whole group, etc.). Some events included multiple codes. When this occurred, each code was indexed separately.

Observations and field notes provided for a more holistic view of classroom setup, instructional supports available to students in the room, and student activity throughout the period. Although student behavior was not explicitly a variable of interest, it was important to understand how the teacher made instructional choices in response to student needs. A visual check of the room also provided data about which classroom norms and expectations were communicated and reinforced (research question 1). Documents were also reviewed for alignment with other evidence sources related to classroom norms, routines, and expectations for

discourse (research question #1). Environmental notes from classroom observations were recorded, coded, and indexed in narrative fashion as explained above.

The teacher was openly engaged in the process of “deliberate analysis” (Brown & Coles, 2012) relating to all data and artifacts collected. Bi-weekly interviews were used both to share observations from classroom observations and to engage the teacher in interpreting the intended purpose of instructional choices. The interview process was used to verify and interpret data related to each of the three research questions.

Teacher interviews were recorded so that data from the interviews could be analyzed. The teacher's reflections on his instructional decisions and the intended connections to SMPs were of particular interest when the interviews were analyzed. Here too, analysis occurred soon after each individual interview to consistently identify emerging themes and to use future observations and interviews to validate the emergence of patterns. In addition to serving as a form of member checking, engaging the teacher in deliberate analysis also offered another perspective on the data which served to promote accuracy within the description of events, awareness of alternative explanations, and consideration for various interpretations.

### **Limitations and Delimitations of Case Study Design**

While the case study methodology allowed the researcher to focus on teaching strategies and conditions for the use of discourse related to SMPs, analysis of the impact of such actions on students' mathematical achievement was beyond the scope of this study. Instead, the findings of previous research (e.g., Attard, 2012; Hannula, 2012; Martin et al., 2015) provided the connection between student engagement (particularly through discourse) and achievement in middle school mathematics.

In this study, data was collected from several classes taught by the participating teacher. Qualitative data collection focused on instructional strategies observed and teacher interviews to explain possible variations in the activities and amount of discourse between groups. Data analysis did not address the *impact* of variations. While it is acknowledged that these may be interesting areas to investigate, a detailed review of variations and impact was beyond the scope of this study. The intent of the study was to examine ways that the middle school mathematics teacher implemented discourse to engage students with the eight Standards for Mathematical Practice within the bounded system of one middle school teacher's instructional activities.

While Yin (2014) acknowledged that generalizability is a potential limitation of case study research, he countered this potential threat by emphasizing strict adherence to quality control measures. Similarly, Stake (1995) recognized the impressionistic and subjective qualities within qualitative interpretations. To help address concerns about validity and generalizability, Stake suggested that case study researcher explicitly attend to triangulation. In this study, triangulation was utilized to achieve quality control through iterative analysis of interviews, observations, and document review.

Another potential issue related to generalizability may be found in the limitations of the setting in which this study occurred. While Merriam (1998) explained that the boundaries of the specific case help to define the case study, it is necessary to understand these boundaries as delimiting factors in the case study. The teacher in the proposed study was one of only two mathematics teachers at a small, rural middle school serving a student population of 98% white and 100% native English speakers. In light of the greater diversity that exists in many other middle schools throughout the country, the population of students taught by the teacher in this study may draw concerns about generalizability. Furthermore, the focus of the case study on one

single teacher is an important delimitation. Although this may appear to be a narrow focus, the analysis of the factors affecting the teacher's choices about how he guided student discourse may lead to some generalizable findings about reflective practice.

This case study focused on the instructional decisions implemented by the teacher as he endeavored to meet the needs of his students. Interview questions were designed to elicit the factors taken into consideration in the decision-making process of the teacher. This justification is an important perspective within the narrative description. It is up to the reader to determine whether the identified factors are generalizable to his/her context.

Another important delimitation in this study was the decision to utilize a strict qualitative process. Since the purpose of this study was not to evaluate a program, nor to quantify the impact of an instructional strategy, but rather to investigate a complex social process through careful analysis of a specific case, the case study methodology was considered appropriate. Careful presentation of the findings and interpretations “provides the reader with a depiction in enough detail to show that the author's conclusions 'make sense'” (Merriam, 1998, p. 199). Merriam reminded her readers that much can be learned from careful examination of a particular case. Similarly Stake (2005) pointed out that researchers,

pass along to readers some of their personal meanings of events and relationships -- and fail to pass along others. They know that the reader, too, will add and subtract, invent and shape -- reconstructing the knowledge in ways that leave it . . . more likely to be personally useful. (p. 455)

As such, generalizability and interpretation of data is as much the responsibility of the *reader* of a case study as it is the responsibility of the author of the study. While readers will likely construct their own meaning from the data presented, the researcher also has a responsibility to

propose explanations and interpretations based on intimate knowledge of the case and prior research on the factors influencing the variables of the case. Such fluidity leads to the possibility of subjectivity in relation to interpretations arising from case study research. As with any inductive process however, the researcher attempted to present enough data to validate and substantiate the claims.

## **Validation**

Maxwell (1996) suggested that researchers consider possible threats to validity when conducting qualitative studies. This proposed study used respondent validation, comparison, and triangulation as means to build a robust and reasonable understanding about the ways that discourse were used to promote student engagement with Standards for Mathematical Practice. Through interviews, the teacher engaged in respondent validation, lessening the potential for misrepresentations or misinterpretations. Respondent validation served to balance the teacher's personal perspective or intention with information observed from lessons and collected through artifacts. Similarly, interviews were used to clarify initial impressions formed by the researcher from observations and document review, through engaging the teacher in a process of member checking. Through member checking, the teacher reviewed themes and categories that emerged through a continuous process of data collection and analysis. Member checking supported the constructivist epistemology as the researcher and teacher worked together to develop and confirm interpretations and conclusions.

Comparison across multiple settings (different classes of students) helped to illuminate possible variables at play. Each group of students was different and their needs as learners varied. Observing the teacher's interaction with students and ways in which discourse-based strategies were used with different classes of students allowed for the possibility of generalizations.

Similarly, Yin (2011) suggested that triangulation improves the validity of observations and interpretations by actively searching for multiple sources of evidence for the event or relationship being described. As such, recorded lessons, researcher observations, lesson transcripts, interviews, and document analysis, coupled with observations across multiple grades and groups of students, offered a well-balanced perspective that aimed to address Maxwell's (1996) concerns about reactivity, descriptive validity, and interpretation validity.

As previously discussed, the use of rich, thick descriptions in narrative format help to establish credibility of the research findings by allowing the reader to determine whether the findings are consistent with the data presented. Multiple sources of data strengthen the reliability of identified themes and study conclusions. Finding from this study should be reviewed against findings of previous studies such as those cited in chapter two. Confidence in the trustworthiness of the findings can be openly judged by the reader because the author has presented a clear, coherent description of the data, its multiple sources, and possible meanings.

### **Expected Findings**

During the 2013–14 school year, the participating teacher was trained to use strategies for engaging students in content-focused discourse. The strategies largely emphasized prompts and protocols for eliciting student 'noticing' and asking students to generate descriptions and explanations. This approach was constructivist in nature and promoted instructional strategies that placed responsibility for noticing and sense-making with the students. Because of the nature of this training, it was anticipated that classroom observations would provide evidence that the teacher elicited information from his students more often than he presented himself as the authority over mathematical explanations.



It was anticipated that SMPs connected to conceptual understanding would be prominent in classroom discourse. O'Connell and SanGiovanni (2013) noted that five SMPs are related to conceptual understanding. The Connected Math series used as the primary resource in the mathematics classes observed was constructivist in nature and presented in such a way as to build conceptual knowledge of mathematics. Because of the classroom text used and the heavy emphasis on conceptual understanding reflected in the SMPs (#3, 4, 5, 6, & 8), it was anticipated that these SMPs would be observed more frequently in relation to the others.

As a consequent of participating in this study, it was anticipated that the teacher would become increasingly aware of the SMPs and intentional in his planning related to inclusion of SMPs as part of his lessons. Through bi-weekly interviews and data review, the teacher engaged in deliberate analysis (Brown & Coles, 2012) and clarified his intended purpose, actions, and outcomes. Because of engagement in this process, it was anticipated that he would become more deliberate in his actions as he considered the outcomes of previous decisions. While the research process was intended to engage the teacher in a process of reflection on pedagogical practices, it was anticipated that the very act of conducting research may have lead to an unnatural focus during the period of the study. Previous discussions with this teacher indicated that he engaged in reflective practice as he implemented instructional changes while reviewing previous videos of his lessons. Through meaningful involvement and participation in the analysis process, it was hoped that any change brought about because of this study would be the result of the teacher's own interpretations, metacognition, and responsive action that co-emerge naturally (Davis, 1999).

### **Ethical Issues**

The researcher was employed as a district level administrator in the school system where this study was conducted. The researcher's role was to work with teachers for curriculum

alignment, improved pedagogy, and effective assessment strategies. It is important to note that the researcher did not have any evaluative capacity within the district.

At the time of this study, the researcher held current certification as a middle school mathematics teacher. This prior experience was useful to the context of the current study as a deep understanding of the conceptual knowledge underlying mathematics processes and reasoning are essential to actively evaluating teacher prompting, efforts to elicit student reasoning, and activities designed to promote student thinking.

As the teacher and researcher had both been employed in the district for four years, they were familiar with one another and had established a professional working relationship. The teacher was familiar with techniques used by the researcher to encourage reflective practice through their regular interactions. When the researcher asked the teacher to describe a lesson or to reflect on data, the teacher did not see this inquiry as a threat or accusation. It is the opinion of the researcher that the teacher was genuinely interested in improving his craft and that he cared about student learning. When asked if he would be willing to participate in this study, the teacher enthusiastically agreed, stating that this project would provide him with much needed time to reflect on his practice – an activity for which he wished he had more time.

The relationship between the researcher and teacher could be characterized as mutually respectful. There was no personal relationship between the researcher and the teacher beyond the school day. In light of the ongoing professional relationship that existed, the researcher worked to maintain a role as a moderate participant observer, balancing previously established rapport with objectivity and inquisitiveness.

Throughout the research period, the teacher and researcher met formally in a content area Professional Learning Community (PLC) with one other teacher on the average of once every

two weeks. As part of the PLC, the researcher occasionally shared articles related to instructional practices in mathematics. These articles typically came from an NCTM publication. A long-term project to align units of instruction to standards, including SMPs, spanned the timeframe of this study. The teacher was working on a system to provide feedback on SMPs throughout the year, formally and informally.

Prior to the start of this study, the researcher had never observed a lesson nor class led by the teacher. In an attempt to minimize the influence of the researcher on the instructional practices of the teacher, it is worth noting that the time frame of this study did not include any in-service days in which the researcher would have been responsible for providing instructional guidance to staff members. Also, during the time period of this study, the principal attended the mathematics PLC meetings as the administrator in charge of guiding decisions. The intent here was to limit formal and informal conversations (not part of the study) about classroom practices that may have influenced the teacher's actions in relation to this study.

As previously stated, there was another middle school mathematics teacher at this school. She was completing her first year in the district. Due to the probationary nature of her contract and her focused work with a mentor, it was not advisable to include her in this study. The teacher identified for this study was free to leave the study at any time. Information that emerged during this study would be shared with the principal only by the initiation of the teacher. The principal and superintendent agreed that no information arising from this study would threaten the teacher's employment with the district nor endanger his career.

## **Summary**

The proposed qualitative case study was designed to examine ways that discourse was used in a middle school classroom to engage students with the eight Standards for Mathematical

Practice outlined in the Common Core State Standards. Observations, interviews, and document collection were implemented to better understand the classroom norms, teacher actions to engage students in discourse about the SMPs, and to monitor the balance of which practices receive attention. Regular interactions between the researcher and the teacher were an important component of this study. These interactions focused on validation of data, coding, and interpretations drawn from the data. The researcher engaged the teacher in reflective practice about the instructional practices he chose to employ and their connections to SMPs. Research prior to this study suggested that adult-mediated discourse in the middle school mathematics classroom positively contributes to both behavioral and cognitive engagement. These are indeed desired attributes of a middle school learning environment and the purpose of this study was to identify discourse-based instructional strategies used to engage middle school students with SMPs.

## **Chapter 4: Data Analysis and Results**

### **Introduction**

The purpose of this qualitative case study was to identify discourse-based structures and practices used by a middle school mathematics teacher to support student engagement with SMPs. The problem that this study addressed was limited to investigation of discourse-based instructional practices to engage students with the eight Standards for Mathematical Practice (SMPs). Publications written for mathematics teachers stress the importance of utilizing student discourse as a means for practicing the skills and dispositions outlined in the SMPs (Stephan, 2014; Thomas, Fisher, Jong, Schack, Krause, & Kasten, 2015). A review of current literature indicated that studies have focused on engagement from students' perspectives using large-scale case studies (Attard, 2013; Darragh, 2013; Way et al, 2015). However, there is a gap in available research describing the choices teachers make to connect discourse to SMPs.

The qualitative case study was conducted at a middle school in a rural district in central Maine. One middle school mathematics teacher served as the subject of the case - the five classes he taught to students in grades six, seven, and eight provided embedded cases within the study. Data collection consisted of classroom observations, interviews with the teacher, and review of documents used during observed lessons. Audio recordings, field notes, and transcripts from the classroom observations and interviews provided opportunities for multiple reviews of the data.

Three research questions informed the purpose of the study and guided the data collection process.

1. How were norms, routines, and classrooms expectations established and reinforced to support student discourse?
2. How were Standards for Mathematics Practice emphasized during instruction?

3. How were teacher-centered instructional strategies implemented to engage students with discourse around SMPs?

Chapter four begins with a review of descriptive data about the case study. The next section includes a description of the data analysis process. The chapter also includes a presentation of the data based on the research questions that guided the study. The chapter ends with a summary of the data analysis and results.

### **Descriptive Data**

To address the purpose of this qualitative case study, data was collected about the teaching practices of one middle school mathematics teacher. The participating teacher was chosen because of convenience (the researcher also works in the same district). The instructional setting, however, is representative of the way mathematics instruction is delivered in rural communities where it is not uncommon that a single teacher would provide instruction to students over multiple grade levels. In the middle school where the study was conducted, the participating teacher provided daily instruction to one class of sixth grade students (19 students), one class of seventh grade students (23 students), and three classes of eighth grade (17, 19, and 20 students per class). With the exception of the eighth grade Algebra I class, all classes were heterogeneously grouped. Table 1 summarizes class data.

Table 1. Number of Observations and Students Present per Class

Class	Observations	Minimum	Maximum	Average
Grade 6	5	17	19	18
Grade 7	4 <sup>a</sup>	21	23	22
Grade 8-1	2	18	19	19
Grade 8-2	4	16	20	18
Grade 8-Algebra	5	15	17	16

<sup>a</sup> Equipment malfunction resulted in loss of data from one of the Grade 7 class observations.

Although the unit of analysis in this case study was the teacher's instructional practices and decisions informing his practice, Merriam (2009) suggests that factors related to the setting of the study help to situate observations within a context. Twenty classroom observations were conducted throughout the first trimester of the school year. The number of students present during observations ranged from 15 to 23 students with an average of 19 students present during each of the observations. The classroom set up consisted of six large tables arranged in groups of two and six individual student desks arranged in a cluster. This was a new arrangement implemented by the teacher for the current school year to promote students working in groups.

The participating teacher used curriculum resources from a variety of sources. The primary resource for all courses except Algebra I was from the third edition of the Connected Mathematics Project (CMP3), a constructivist mathematics program designed for middle school use. Occasionally, freely available materials from Big Ideas in Math and Eureka Math, also known as Engage NY, were used to supplement the primary text. Within the instructional setting, technology was available to students and the teacher. The teacher shared resources with students and reviewed student work electronically through an application called eBackpack that served as an online learner environment. During every observed class, the teacher used his iPad with the interactive whiteboard to project problems and move between applications used for instruction.

Through a statewide technology initiative, all students in grades seven and eight had one-to-one access to an iPad Pro. Students in grades seven and eight were encouraged to take notes and do their work digitally but they were given the choice to work with paper and pencil if they preferred. Students in grade six had access to MacBook laptops. Although class materials were available to sixth grade students through Google Classroom, sixth graders were not observed using their laptops during math class.

Classroom observations were conducted from the back of the room, near the teacher's desk. A few minutes prior to the start of class, the recording device was set up and preparations were made so that note taking could begin as soon as students arrived. During the observations, the participating teacher wore a lanyard with a microphone that recorded directly to an iPad using the Swivl app. Although the hardware and software used for recording had the capability of providing video recording (it was not possible to record the audio without the video), a piece of tape was placed over the device's camera to obscure any images.

The researcher used direct observations to witness discourse events in their context. Nineteen of the 20 classroom observations were captured in an audio recording via a microphone attached to a lanyard around the teacher's neck. The average classroom observation lasted 41 minutes and 30 seconds. More than 13 total hours of audio were transcribed from classroom observations. Transcripts of classroom observations yielded 186 pages of data.

Immediately following each observation, the recording was downloaded from the Swivl cloud. An external digital copy was created and a copy was uploaded to a private YouTube account. YouTube was chosen as a storage site for ease of sharing data with the participating teacher and for the capability the service offers for auto-generating a transcript of the audio within a file.

Within 24 hours of uploading each file to YouTube, a transcript was available with timestamps every three seconds. The transcripts were copied to a word processing document for editing. Within 2 days of the original observation, each transcript was reviewed in conjunction with the audio recording and edited for accuracy. Each transcript was saved in a separate file and a combined transcript file was also created which included all classroom observations. A similar process was used for recording and transcribing each of the interviews.



Five interviews were conducted to probe deeper into the intended purpose of teacher-centered behaviors related to the use of discourse. Over two hours of combined audio recordings were captured from interviews with the participating teacher. All interviews took place in the teacher's classroom either during a planning period or after school and lasted an average of 24 minutes and 29 seconds. All interviews together yielded 21 pages of transcripts and five pages of researcher notes.

Prior to interviews, the participating teacher was provided with electronic transcripts of the observed class (the teacher preferred electronic copies). Four of the interviews followed a semi-structured format using the interview questions included in Appendix C. The original format was implemented to engage the teacher in reflection on a lesson or instructional event. One interview, the third interview, was slightly different in that it was formatted as a pre-interview. Rather than focusing on a lesson that had already been taught, the participating teacher was asked to identify and discuss the types of considerations he had made in preparation for an upcoming lesson - learning target, instructional strategies and student activities, and anticipated student struggle. The lesson he discussed was then taught during the next class period on that day and was observed and recorded. The remaining two interviews followed a reflective practice approach similar to the first two interviews.

In addition to the audio recordings and transcripts, each observation and interview yielded a single page of field notes. Field notes recorded elements identified by Merriam (2009) as important aspects of observational data - the physical setting, participants, verbal prompts by the teachers, context of the class (time of day, day of week, other relevant events), class activities, and initial researcher impressions. Field notes were used to document points of inquiry for future interviews. During group activities the teacher moved around to groups. Due to classroom noise

and activity, it was not possible to capture the nature of table discussions in field notes. While audio recordings and transcripts were used to clarify such interactions captured through the lanyard microphone, field notes were used to record observed phenomena that the audio recording did not capture - e.g., the specific problems with which groups grappled. The combination of audio recordings and field notes from observations provided a more holistic view of classroom setup, instructional supports available to students in the room, and student activity throughout the period.

Documents collected from classroom observations related to the topic of the lesson, student handouts, and classwork or homework assignments. The participating teacher shared these documents electronically, just as they had been shared with students. As it was common practice for the teacher to provide an entire unit to students in the form of a digital packet, many of the documents shared included resources that were not part of the observed lessons. These documents provided background about lessons and prior learning experiences students had leading up to the observed lesson.

### **Data Analysis Process**

Multiple qualitative data analysis techniques were used to reduce 186 pages of classroom observation transcripts and approximately 21 pages of interview transcripts into manageable units related to the research questions identified. Using a process outlined by Merriam (2009), highlights and notations were made within the text next to bits of data that stood out as relevant to the research questions. Standards for Mathematical Practice were identified in the transcripts and recorded in a spreadsheet using codes specific to each of the eight SMPs. Open codes were created from the highlighted terms associated with classroom expectations, norms, and routines. All open codes were catalogued in an electronic index and entered into the analysis spreadsheet

for future sorting. Later, open codes were grouped by common themes and axial codes were generated in a new column of the spreadsheet to highlight the commonality (Merriam, 2009). Descriptive axial codes and category labels were chosen to reflect the language and emphasis of the teacher.

Throughout the course of data collection and analysis, discourse-based structures were reviewed multiple times: at the time of the initial observation; during the initial transcription process; prior to each interview; during data reduction for evidence of Standards for Mathematical Practice; during data reduction for evidence of norms, routines, and classroom expectations related to discourse; and again in part and in whole during the process of solidifying themes and categories. During the process of creating the initial transcripts, annotations were added to the right-hand margin for future reference. Such notes included references to the eight SMPs and questions for future interviews. Codes were also added to mark areas within the transcript where the teacher sought to establish or reinforce expectations for student discourse. Incidents in which classroom expectations were communicated became topics of conversation during interviews to determine whether expectations (either stated explicitly or implicitly in the classroom) were captured accurately.

The circular, progressive nature of data analysis simultaneous with data collection is a defining characteristic of qualitative data analysis (Merriam, 2009). As more codes developed within successive transcripts, previous transcripts were reviewed to test the consistency of emergent themes. Internal validity was enhanced through the ability to verify themes that emerged from transcripts against researcher impressions captured in field notes, and through member checking conducted with the teacher via interviews. The process used for this study is supported by Yin's (2015) suggestion that the reliability of verbatim transcription juxtaposed

against the subjective and impressionistic nature of field notes and interviews are necessary features in validity of case study research.

Following the final observation and interview, the combined transcript of all observations was again reviewed in its entirety. The purpose of this first post-observation review was to reduce the relevant data for each of the eight SMPs into a table format. A spreadsheet was created to document the location and context of SMPs throughout the transcript. Initially, the spreadsheet consisted of five columns. The first column captured the SMP reference by number (1–8), the second column indicated the date of the observation, the third column recorded the class in which the incident occurred, the fourth column listed the time stamp within the transcript, and the fifth column included notes about the context of the incident in which the SMP was identified. Nearly 150 episodes were initially identified in which the teacher specifically sought to engage students in practices associated with mathematical proficiency.

Another review of the entire transcript of interviews and classroom observations was conducted and norms, routines, and classroom expectations were coded. Substantive words and locations within the transcript were indexed for future review using the indexing function within the word processor. After initial indexing was complete, terms were reviewed for sorting and category coding. The index was used to identify themes within the data with multiple references. The index also provided quick access to specific scenes in the transcript for deeper review. Next, scenes within the transcript that provided evidence of classroom norms, routines, and expectations for discourse were entered into a new spreadsheet with similar column headings as described for SMP identification.

The combined spreadsheet consolidated evidence from the transcripts that identified SMPs and with evidence regarding classroom expectations and norms. Within the same

spreadsheet a new column was added to record the type of instructional activity designed by the teacher to serve as the delivery mechanism for the SMPs and/or expectations. Transcripts from interviews with the teacher, field notes from interviews with the teacher, and personal judgment about the type of activity lead to the development of instructional activity categories. To address the purpose of identifying discourse-based structures and practices used by the teacher to support student engagement with SMPs, the combined spreadsheet was used to capture patterns that emerged between SMPs and teacher-centered instructional activities.

## **Results**

The following results are presented as a means for understanding the role of discourse in mathematics instruction within the context of the middle school mathematics classroom where the study was carried out. Specifically, to address the purpose of this study, data are presented to identify discourse-based structures and practices used by a middle school mathematics teacher to support student engagement with the eight Standards for Mathematical Practice (SMPs). The following research questions helped to focus the data collection and analysis process. Findings are presented in the following section to address the questions in order.

R1. How were norms, routines, and classroom expectations established and reinforced to support student discourse?

R2. How were Standards for Mathematic Practice emphasized during instruction?

R3. How were teacher-centered instructional strategies implemented to engage students with discourse around SMPs?

Themes were identified for each question from the episodes recorded in the combined spreadsheet. While the initial development was done using an inductive process, later columns were added to ensure that each research question was addressed thus applying a deductive

element. Thematic naming was a combination of teacher terms used during instructional activities and research terms used to describe the activities taking place.

**R1. How were norms, routines, and classroom expectations established and reinforced to support student discourse?** Research question one required that data collection begin on the first day of school. Seven observations were conducted in a total of four different classes prior to the first interview. During those first seven observations and continuing throughout the observation period, attention was given to specific classroom expectations communicated to students, routines for student participation, and norms for student interactions. Interviews were used to elicit feedback from the teacher about expectations for student interactions, the teacher's role during activities and discussions, and whether the intended goal had been achieved.

*Thematic category 1: Discourse as a means of practicing metacognition.* The teacher worked to build student understanding and establish routines for metacognition. Beginning in the first week of classes, he used activities and led discussions to help students focus on strategies and to identify characteristics of effective and ineffective strategies. During the first meeting with the grade eight Algebra class, the teacher set out this expectation:

What we're talking about when we talk about metacognition, the easiest way to think about it is to just . . . that you are thinking about how you are thinking. Okay? You're thinking about what your brain is doing to produce information. Kind of thinking, 'Okay, how did I do that?' because when you look at something, your brain automatically starts to pick up on patterns, right? So one of the things we want to start doing in algebra is, you want to start thinking about, 'Okay, what is my brain doing? What am I

automatically locking on? What stands out to me? Like what am I recognizing that is familiar?’

The focus on metacognition was seen in several instances when the teacher asked students to identify and reflect on the strategy they used to approach a problem. Sixth grade students were asked to think about the strategy they used while playing a round of the Product Game. Prior to the activity, students were prompted:

When you're just starting a new game, what I really want you to do, is I want you to really start looking at how the numbers are placed on the board. What groups of four or lines of four would make the most sense to go for? So what numbers are going to be easy to get for you? What numbers are going to be difficult to get?

During the activity, the teacher moved around to groups and prompted them about their strategy. After 10 minutes of student activity, he brought the class back together for a discussion:

What I want to do is have a quick conversation about what you saw in the game. What numbers were easy to get and what numbers were harder to get? Any strategies that you guys used? . . . So strategies, who had some strategies you thought worked really well?

During the discussion that followed, a number of strategies emerged. More significantly, students discovered that some were explaining the same strategy (blocking) from a defensive perspective while others were explaining it from an offensive perspective. The teacher reflected on this lesson and the resulting discussion during a subsequent interview:

The more group stuff we did, the better variety of discussion pieces, I guess you could say, we had. Like there is the different definitions [*sic*] of blocking that we were talking about. . . . There was more of a variety of strategies and not necessarily just a variety of strategies, but that one strategy was explained from a different perspective. Even though

it was in general the same strategy, having it explained from a different perspective - if I would have explained it, I would have explained the strategy the same way every time. But the students, two students explained the same strategy using different words or different phrases.

During the twelfth week of classroom observations, students were struggling with a ratio problem. The teacher used a verbal prompt to scaffold their thinking about a particular strategy, “When I was walking around I saw this, where people are separating it into Saturday information and Sunday information. Why might that be a good strategy? Why might that help us?” Through verbal prompts and intentional use of classroom discussions, the teacher set forth an expectation that different students think about and approach mathematical problems differently. Students were given opportunities to think about their own thinking and to verbalize their thinking. The expectation of metacognition was established early in each class and students were expected to practice it often.

***Thematic category 2: Notes as a resource for discussion.*** From the first meeting with each class, the expectation that students would take notes and the process for how their notebooks should be set up was shared. The word notes appears 89 times in the transcript of classroom observations and 49 times in the transcript of interviews. Much of the third interview was spent uncovering the purpose of notes from the teacher’s perspective.

During early classroom observations, it appeared that notes were a procedural part of classroom routines. During the first meeting with seventh grade students, the teacher explained to students their responsibility for taking notes:

You're going to have two things you are really responsible for - one is homework which you guys are familiar with, the other one is going to be class work and notes. Class work



and notes can go together but we need to make sure we keep them separate from homework.

Classes often began with a reminder to take notes, “Go ahead and take out your notes. We're going to take some quick notes and then we're going to do an activity.” Organization of notes was enforced, “Make sure you have a specific place for your notes ... Remember, our class work this year is part of our notes. So what we do in class, take that down, right after your notes.”

As lessons progressed, the role of notes as a resource for group work, classroom discussions, and independent work began to emerge. As students began working together on practice problems, they were prompted to refer to their notes, “If we're not sure where to start, what should we probably be doing? ... go to your notes.”

A grade seven lesson on adjacent angles illustrated the expectation that students use their notes first to think independently about a problem and then to discuss their thinking with peers. In this case, the notes were a resource to support student conversations, justify decisions, and reach consensus:

Look in your notes if you don't remember what adjacent angles are. We took those notes Friday. Once you have done that, within your table group, what I would like you to do is, I'd like you to discuss what you came up with for adjacent angles. If you came up with the same ones. If you came up with different ones. If you agree or disagree with what the other people came up with. So once you have your adjacent angles go ahead and have that discussion.

Through the use of notes, the teacher worked to establish independence:

If you come to a group of questions that you don't understand, my expectation is, you're going to show me - somehow - that you made an effort to solve the problem. Whether that's writing a sentence; I looked in my notes and I found this definition . . .

During classwork, groups worked together to make sense of problems and work toward a solution. As the teacher moved between groups, he prompted groups to use their notes as a resource without always relying on the teacher to direct their learning:

I'm having a lot of questions and kind of giving the same answers over and over again. ...

You have all kinds of examples from before that you can look back at. We have all kinds of notes. ... We need to start being a little bit more independent with it.

## **R2. How were Standards for Mathematical Practice emphasized during instruction?**

During classroom observations, initial impressions of SMPs were recorded in the form of field

Table 2. Breakdown of Standards for Mathematical Practice (SMPs) Observed by Class

SMP	Total	Gr 6	Gr 7	Gr 8-1	Gr 8-2	Gr 8 Alg
1 – Make sense of problems and persevere in solving them.	24	7	3	0	6	8
2 – Reason abstractly and quantitatively.	10	3	0	2	1	4
3 – Construct viable arguments and critique the reasoning of others.	50	18	7	2	11	12
4 – Model with mathematics.	4	3	0	0	1	0
5 – Use appropriate tools strategically.	3	0	3	0	0	0
6 – Attend to precision.	17	5	4	1	4	3
7 – Look for and make use of structure.	23	5	6	1	4	7
8 – Look for and express regularity in repeated reasoning.	8	4	0	0	2	2
	139	45	23	6	29	36

notes. During transcription, incidents were noted in the margin where evidence of SMPs occurred. Following transcription, field notes were reviewed and incidents originally noted were given a more thorough review as instructional activities and teacher expectations were identified for each incident. Although it is possible for a single incident to be used as an example of more than one SMP, each incident was coded for the primary SMP of focus.

One hundred thirty-nine incidents of SMPs were documented within the transcripts from 19 classroom observations (Table 2). The purpose of R2 is to understand how SMPs are emphasized during instruction. Therefore, analysis of each event was considered from the perspective of the teacher's actions related to the SMP. For example, incidents related to SMP 1 (Make sense of problems and persevere in solving them), were identified based on the emphasis the teacher placed on sense-making and perseverance, not whether the students exhibited these traits.

After all incidents of SMPs were identified, they were coded for instructional emphasis. Forty codes were generated in the first round of coding. Then, the codes were categorized into four themes. Table 3 shows the highest frequency codes within each category. Themes are not exclusive to a single SMP.

***Thematic category 1: Perseverance.*** Although perseverance is specifically connected to

Table 3. Themes identified from high frequency instruction emphasis codes

Perseverance	Express thinking	Use precision	Reach consensus
Make sense of process	Explain process	Clarify	Apply criteria
Make sense of solution	Identify common	Communicate with	Apply rule
Reread	mistake	mathematical	Demonstrate reasoning
Try a different strategy	Explain why	symbols	
	Justify with an	Use correct	
	example	terminology	
	Judge others' rationale	Represent quantity	

SMP1 (Make sense of problems and persevere in solving them), perseverance was evident in other SMPs as the teacher asked students to make sense of processes and solutions, reread for understanding, and to try different solutions strategies. During a seventh grade geometry lesson on angles, students had worked in groups to determine the size of various angles using protractors and angle rulers (SMP5 – Use appropriate tools strategically). While students worked, the teacher observed students' solutions. During the class discussion that followed, one student shared his groups' measure for angle CVB. The teacher emphasized perseverance while also asking students to critique the reasoning of others (SMP3), "Who's not sure what Grant's talking about right now? He said CVB and he's estimating it at like 240 degrees. . . . Think about what he said though. What angle is he talking about?"

As students in eighth grade were introduced to exponents and square roots, the teacher used prompts to help students engage by identifying familiar structures (SMP7 – Look for and make use of structure), "The square root of 3 times the square root of 3. Who can give me a place to start? I just need a starting point." As students struggled to identify a way to calculate the square root of three, the teacher provided a scaffold by introducing a more accessible problem using perfect squares, "Let's try to work with some numbers that are easier to work with. Let's say 'the square root of 9 times the square root of 9.'"

The eighth grade square root lesson was later discussed in an interview. The teacher explained his hope that helping students identify a starting point would aid them in persevering through incrementally more complex problems. He sought to help students discover the rule by working through analogous problems, "So really, what they are doing is they are seeing enough examples so they get the rules and then we can infer the rule on strange problems like the square root of 3 times the square root of 3."

One week later, the teacher emphasized perseverance again when he worked with the same eighth grade class to flexibly maneuver terms within an equation. Students had been using the Pythagorean Theorem to find the length of the hypotenuse of a right triangle. After several lesson in which students became familiar with the equation and were able to substitute measurements into the equation from a diagram (SMP2 – Reason abstractly and quantitatively), the teacher was ready to extend the lesson to have students find a missing side length. Rather than providing the equation in a different format, the teacher asked students to cognitively engage with the content by having them compare samples:

Look at this and come up with, what is different about this example from the examples we worked with before? . . . What makes this example different from what we worked with before? Once you come up with that, I want you to share your idea with the group. So share your ideas with the table group - see if they came up with similar ideas.

During whole group instruction, the teacher worked through an example with students that they entered into their notebooks as a model. Then, they worked through a similar problem on their own to try out the process of solving for side length  $b$ . During the discussion that followed, a student explained the steps he used for squaring the known side length and hypotenuse and subtracting the values. The teacher stopped the student's explanation to ask, "Why do we subtract? So, we know you take the square root of that, right? But why do we subtract those?" After the teacher was satisfied that students understood that subtraction was needed when solving for side lengths, he used mathematical terms to name the process (SMP6 – Attend to precision) and made the connection to previous content:

So, that's the inverse operation that we've been working with for a few years. Right? And that helps you when we get down to  $b^2 = 625 - 225$  and we subtract it and then, as [Bob] said, take the square root to give us  $b = 20$ .

While focusing on mathematical concepts, the teacher used examples, questions, and small group discussions to keep students engaged in the content. He recognized when the concepts were more complex or the strategies more sophisticated and he fluctuated between simpler, more familiar problems to give each student an entry point into the problem. Throughout the instructional incidents where SMPs were identified, perseverance was recognized as an emphasis in connection with all SMPs except SMP4 (Model with mathematics) and SMP5 (Use appropriate tools strategically) which both had very small samples.

***Thematic category 2: Express thinking.*** Students were expected to express their thinking in relation to all SMPs except SMP5 (Use appropriate tools strategically). The teacher emphasized oral expression of ideas in 40 incidents related to SMPs. Students were asked to express their thinking by providing their own explanations and justifications, and judging the validity of others' statements.

In an interview, the teacher identified the importance of students expressing their thinking:

I think talking with them about kind of how you discuss things . . . I think that kind of expectation or norm helps out a lot. Making sure kids don't just say you're right or you're wrong. We've had a lot of talks about that, you know, why do you think this way. I think that has added a lot to the group discussion.

Twenty-three out of 40 identified incidents within this category were connected to SMP3 (Construct viable arguments and critique the reasoning of others). Students were frequently

asked to reflect upon the thinking of their classmates as they constructed their own mathematical understanding. In the sixth grade lesson on ratios, students were using tape diagrams to model relationships (SMP4 – Model with mathematics). The teacher asked for an explanation and then asked students to reflect upon that explanation, drawing attention to the way that quantity is represented in the model:

Since we have a huge number like 192, is that going to change our tape diagram?

[Student responded] What do you mean, the number inside? I'm going to have someone else answer. So George says the number inside, but not the tape diagram. We have a huge number - 192 is a pretty big number, much bigger than any other number we've been dealing with here. Why is that not going to change the tape diagram? And what does Gary mean, it's going to change the number inside the tape diagram? What does that mean?

The teacher drew students' attention to the quantity represented by each box of the tape diagram without providing the explanation for them (SMP2 – Reason abstractly and quantitatively). Later in the class, students worked independently on a new ratio problem. The teacher used the previous example from above (for every 5 cars registered, there were 7 trucks registered) to provide reteaching to a student who was struggling with the tape diagram representation:

If I had 12 vehicles, how many vehicles would be represented by each of those parts?

[Student responded] If I had 24, how much would each of those parts be worth? [Student responded] What about if I had 36? [Student responded] How are you getting those numbers? [Student responded] Can you use the same process to figure it out for 192 vehicles?

Through brief questions, the teacher supported the student's ability to express his thinking. Using a simpler form of the original problem, the teacher enabled the student to gain access to the ratio (SMP1 – Make sense of problems and persevere in solving them). By repeating the process for successively larger quantities, the student was able to recognize the pattern within his reasoning (SMP8 – Look for and express regularity in repeated reasoning).

In an eighth grade lesson designed to reinforce combining like terms, students were asked to first make observations about two expressions ( $x^3 \cdot x^2$  and  $5x \cdot 2x$ ). Then, the teacher asked students to agree or disagree with explanations presented, “Who agrees with Carl's observation - the top two [ $x^3 \cdot x^2$ ] have exponents, the bottom two x's [ $5x \cdot 2x$ ] don't have exponents? Does anyone disagree with that observation? Inez, why do you disagree?”

When students' responses were incomplete, the teacher offered further prompts to assist students in recognizing the misconception:

So he's saying this variable has an exponent [ $x^3$ ], this variable has an exponent [ $x^2$ ], this variable doesn't have an exponent [ $2x$ ]. Chloe? [Student responded] What do you mean, one? [Student responded] What exponent? [Student responded] The  $2x$ . Ok, so Chloe is saying this exponent is an exponent of 1. What do you think? There's no exponent there. With intentional questioning and an explicit example, the teacher was able to orchestrate a discussion in which students expressed their thinking about exponents.

Small group disagreements were also used to initiate whole group discussions. Part way through an Algebra class on functions and relations, students were given eight problems (tables, graphs, and equations) and asked to work in groups to decide whether each represented a function and why, based on previously established criteria (SMP7 – Look for and make use of



structure). Recognizing that one item generated considerable discussion, the teacher brought the problem to the whole class:

So let's look at E. We're having a discussion on whether or not it's a function. So Carrie was saying we have two 6s. Let's look at it again. So we have two 6s. Ok, but Gina is saying she disagrees. What do you think? [student states that he agrees with Gina] Why Nicholas?

Employing SMP3 (Construct viable arguments and critique the reasoning of others), the teacher asked students to consider Carrie's argument that having the same output (6) for two inputs means that it is not a function. In order for students to agree or disagree, they first needed to understand Carrie's argument. In setting up the discussion, the teacher used mathematical terms of importance – input and output. The discussion led another student to question his group's decision on a previous problem (D) that the teacher was able to use to extend the conversation to clarify inputs and outputs:

Mark is asking a good question. ... So Mark is trying to compare these two right now. He says, 'Well, I say that I can have two sixes and it's still a function. But up here, I said it wasn't a function.' What's the difference, Paul?

Throughout the observation period, the teacher asked questions and led discussions that required students to express their thinking by explaining processes, justifying their explanation, and judging others' rationale. Mark's question shows that students used similar techniques as they monitored their own understanding and attempted to make sense of problems.

***Thematic category 3: Use precision.*** Precision was evident in all SMPs except SMP4 (Model with mathematics) and SMP8 (Look for and express regularity in repeated reasoning). Precision was emphasized by the teacher's expectation that students clearly represent quantity

through the use of correct mathematical terminology and mathematical symbols, and that students provide enough detail to express mathematical ideas clearly. Thirty examples of precision were identified in relation to SMPs.

Precision was emphasized throughout the geometry unit in seventh grade. As students represented angles, several prompts helped students attend to precision:

You said, 'angle CA?' We're missing something. What are we missing? [Student responded] CVA. Now, we know that C and A are the endpoints of our rays, right? We talked about rays and how when two rays meet they create an angle. What is V considered? It has a specific name? [Student responded] Vertex. V is the vertex of the angle.

After some vocabulary work, students applied their definitions to quickly identify angles as acute and obtuse. Then, they calculated missing angles based on the definitions of complementary and supplementary. As students worked in groups to measure and describe various angles, one student realized that the protractors were different which led students to confusion about angle measurements. The teacher mediated a discussion that led to clarity based on previous understanding of angles:

You have two sets of numbers. Right? You have two sets of numbers on your protractor. If you have an angle ruler, you don't have to worry about it. With yours, you have two sets of numbers. So if I'm looking at an angle AVB, okay, and I get 150 degrees, what does that tell me I'm doing? So if I'm looking at AVB and I say I have 150 degrees, Melanie says that I'm wrong - 150 is over here. Okay. [Student responded] Why is it too high? [Student responded] Here we go, because it's an acute angle.

The teacher was able to clarify the confusion about the different protractors (SMP5 – Use appropriate tools strategically) by having students reason about their measurements (SMP2 – Reason abstractly and quantitatively) based on the definitions of terms.

Students were expected to begin problems related to the Pythagorean Theorem by writing the equation out and then defining each variable:

What's the first thing I want you guys to do, to make sure we don't make any mistakes?

... [Student responded] Okay, we can write out the Pythagorean Theorem. . . . First of all, replace the variables with the numbers that you know they equal. Right? So write that stuff out - 15 squared plus 'we don't know what b is' equals 25 squared.

As students worked through the problems, they often forgot the last step. While helping students to remember the last step, the teacher also focused on precision by guiding students to clarity about the distinction between squaring and finding the square root, “When you say you forgot to square it, what are you talking about? You forgot to . . .”

The expectation for precision when communicating about and representing quantities was evident in the prompts used by the teacher. While working in groups, students in sixth grade were reminded to use labels to precisely refer to quantities within a ratio:

We're not just saying numbers. If I say 45 to 18, do you guys have any idea what I'm talking about? No. When I say 45 to 18, there's no context. Give it some context and when you say your numbers, follow it up with whatever that number refers to.

Group discussions were used to elicit ideas about the meaning of quantities and the ways they were represented. After creating a tape diagram of a ratio as a whole group, students were given a specific topic to discuss as they worked to make sense (SMP1) of the importance of equal-sized groupings:

Part C is asking how many equal size parts does the tape diagram consistent of? How many equal-sized parts do we have? How many equal size parts, Gil? [Student responded] 12 - we have 12 equal size parts. So now, what I want you to do is, in your group, I would like you to come up with a description of what that 12 stands for. . . . come up with the description in your group of what that 12 represents.

The teacher chose student discussion topics and used prompts that emphasized precision as students were expected to clarify their thinking and communicate about quantity using correct symbols and terminology.

***Thematic category 4: Reach consensus.*** Twenty-eight incidents were identified in which the teacher created opportunities for students to work together to make decisions related to application of mathematical rules. Eleven of the twenty-eight episodes were identified during instructional activities related to SMP3 (Construct viable arguments and critique the reasoning of others). In some cases, students worked together to generate explanation; in other cases, students responded to explanations generated by classmates. The teacher often framed consensus episodes in terms of agreement or disagreement, but it was also evident that a defense of judgment was also expected.

During an interview, the teacher expressed his expectation for the types of interactions students should be having during group work and small group discussions:

Usually what I like to listen for is people saying, 'Well I got a different answer. This is my answer,' and not kids saying, 'Oh you got that answer? I'm gonna write it down.' So I'm looking for kids to be having that conversation. Because obviously with something like this, there's a bunch of different answers that you could come up with. Also looking for kids to, ... asking for clarification.

Although consensus, or group agreement, was often stated as a goal, the teacher was prepared in case consensus could not be reached. By monitoring students as they worked in groups, the teacher was able to judge when additional teaching was needed and was also able to extract multiple strategies to solutions:

So this group went about it in a completely different way. And that's really one of the things that I wanted this to bring out, is the fact that, in math, you don't always have to - you really don't always want to approach a problem the same way over and over and over and over again. ... Now that we've heard a few of these strategies, I want you to see if you're going through and you're using a strategy that's not really working for you. Can you adjust it? Adjust the strategy. Try! Maybe go about it a different way.

In Algebra, students were given a graph and asked to work in groups to explain why the graph represented a function and then develop two additional ways to describe the function (SMP7 – Look for and make use of structure). Students were expected to apply criteria captured in their notes from previous days' examples as a resource to develop their explanations. While the teacher moved between groups, he offered supports to build independence without removing productive struggle, “Once you have an explanation as to why it's a function, move on to describing it two other ways or representing it two other ways. ... How's it going here? Go back to your definition of function.”

In seventh grade, students worked in groups to sort shapes based on attributes of the shapes and then document the criteria for membership. After a few minutes of sorting shapes, the class was called together to share their criteria. Based on criteria shared, the teacher highlighted a nuance in two different explanations for sorting the trapezoid, “First definition: a trapezoid is a quadrilateral with one set of parallel sides. That's one definition. The other

definition: a trapezoid is a quadrilateral with exactly one set of parallel sides. What's the difference there?"

After drawing students' attention to precision in the criteria for sorting shapes, he brought them back to criteria shared by two groups. Students were asked to make a decision based on precise criteria, "Go back to Rachel's criteria, at least one 90 degree angle. So if Josh's criteria is only one 90 degree angle, would all of Rachel's shapes fit into that group as well?" The lesson was further extended through a series of activities in which students created their own criteria and used the criteria to create and judge new shapes together:

Make it a really exclusive group. Ok? The first part is, describe the property shared by the members of the group. And the second part is to sketch another shape that belongs to the group. This first part, describing it, I would like you to do together. The sketching part, what I would like you to do is, initially, I want you to sketch it separately. And then, what you're going to do is, you're going to come back together and talk about whether or not you agree that everyone's sketch fits into that group.

Throughout this series of activities and discussions, seventh grade students created and applied criteria and developed consensus as they demonstrated understanding of the concepts that were targeted.

When students struggled to find an entry point to problems, the teacher often relied on small group discussions to set up an environment where students had to be clear about what they knew and try various strategies to make sense and persevere through the problem (SMP1). In one Algebra class, many students had struggled on some homework problems. When they encountered challenge, many students skipped the difficult problems. The teacher asked students

to work in groups and assigned one challenge problem to each group to solve together and put their work on the board. One group was assigned a problem that they had each skipped:

Why don't you guys take 26. [student stated that none of the group had done that problem] Perfect! ... The first thing you need to do is, in your group, you need to discuss how to do the problem, what the common mistakes would be, and if you've got the right answer. Ok? Once your group is, has all agreed that you're ready to, go up to the board and kind of spell this out for the class.

The problem was an incorrect worked example in which the students had to figure out the mistake and fix it. The students in the group were unable to identify the mistake and rewrote the problem and the incorrect steps on the board as presented in the original problem. A five-minute discussion followed in which the teacher prompted students to explain each step in the solution until the error was identified:

So we started out by distributing on each side of the equation. [student response] Ok. So  $12y$  equals  $12y$ . Is  $12y$  going to equal  $12y$ ? [student response] Yeah. Somebody else from that group. [student response] What do you mean, they subtracted instead of divided? [student response] Ok, you would divide. So what's going on here?

**R3. How were teacher-centered instructional strategies implemented to engage students with discourse around SMPs?** Teacher-centered instructional strategies fall into three main categories: creating opportunities for small group discussion, providing time for independent thinking before engaging students in discourse, and use of purposeful questioning. Although questioning strategies were used in connection with small group discussions and independent thinking, the evidence presented reflects primary emphasis on the thematic category. SMPs are noted to make the connection clear.

***Thematic category 1: Small group discussions.*** The table group arrangement of the class supported quick movement in and out of group discussions. Small group discussions were often orchestrated so that students would need to construct viable arguments and critique the reasoning of others (SMP3). In grade six, the teacher asked students to turn to their small groups to identify known types of ratios and to begin contemplating other types of ratios that might exist (SMP1 – Make sense of problems and persevere in solving them):

So, what I want to do real quick in our groups, is I want you to just talk about what ratios we know of, and how we are going to create different ratios. So go ahead and have a real quick discussion on how you would create four different ratios using those numbers.

In another grade six lesson, students reviewed factors and multiples by playing the Product Game. After playing the game for a few minutes, the teacher asked them to stop playing to have a discussion about their observations and strategies:

What I want to do is have a quick conversation about what you saw in the game. What numbers were easy to get and what numbers were harder to get. Any strategies that you guys used. ... Go ahead and have a quick table discussion.

Such conversations required that students construct viable arguments (SMP3).

After an introductory lesson on exponents, students in eighth grade were asked to work in groups to collaboratively generate a rule (SMP7 – Look for and make use of structure) based on the example  $2^3 = 2 \times 2 \times 2 = 8$ :

So the number here [2] is my base. This is my exponent [3]. So this is what I'd like you do ... just real quick, with somebody near you in your group, what I would like you to do is, I want you to come up with 'what the exponent does' just based - just looking at that.



After a quick refresher, students in the other eighth grade class were asked to work in groups to come to agreement about how to simplify expressions, “So what I want you to do is finish those next three in your group. Make sure your group agrees on what you're doing. How would I simplify these? Go ahead, do that please.” Several days later, students had worked with rules for simplifying and computing with exponents in different situations. Having presented students with a variety of exercises which represented the various situations for using with exponents, students were asked to work in groups to compare solutions based on knowledge of the rules, “So, what we're going to do now, in your groups, I would like you to compare your answers to these. Talk about what rules you used. If you got different answers, talk about why.”

***Thematic category 2: Opportunities for independent thinking.*** To engage a variety of students, the teacher provided opportunities for students to consider the context of problems, try a strategy independently, or begin making sense on their own before entering into groups (SMP1 – Make sense of problems and persevere in solving them). When working with functions, Algebra students tried some problems on their own before they discussed them in their groups, “You guys are going to do it on your own first and then discuss it.” Weeks later, the Algebra group began working with the Pythagorean Theorem. The teacher reviewed several examples with the class as students recorded the examples into their notes. Then, before working together in groups, students were expected to work through a problem independently, “I'm going to give you guys a shot to do one on your own.”

The presentation of the sixth grade unit on ratios was heavily text-laden in which problems were presented as extended scenarios in context. After having worked through an example together, the teacher laid out a process for having students consider the entire problem.

Independent thinking was required of students before they met with their group to begin working toward a solution:

We're going to read it as a class and then we're going to think about it independently.

Then, we're going to come to our groups and share kind of what we've started. Okay? ...

Let me go through it again. We're going to read it as a class. Then, independently to start with, we're going to try to find where would we start to solve this problem? What would we start by doing? Initially, when we come back as a group, we're just going to share our starting point.

By creating a situation where students were able to identify a starting point, the teacher helped students make sense of problems and persevere in solving them (SMP1).

During an interview, the teacher shared his frustration with off-task behavior in the seventh grade group:

They really struggle working in groups. It's one of those groups that is consistently off task. It's very difficult. They end up either bickering or just getting goofy. Consistently. I'm having trouble with them working in groups. And we're going to do a little bit of group work today where we're going to talk about vertical, adjacent, complimentary, supplementary, and then introduced corresponding angles.

He related off-task behavior to ineffective group work in which group members relied on one or two people to do all of the work:

I think that's part of the part they are having trouble with - they come to the group and they're still just kind of looking at the one or two kids that they know typically have a clue about what's going on and going, 'Okay go ahead and tell me.'

In an attempt to maintain the value of group discussions, the teacher decided to implement a strategy that required students to begin engaging with their work independently prior to joining with their group:

I think I'm going to try to kind of piece the group work in, kind of almost like a think-pair-share kind of thing. Where I'll have them work individually first ... where, on your own, answer this and compare it with your group's answers.

Independent practice was provided to give students an opportunity to initiate their thinking as they worked to make sense of problems and persevere in solving them (SMP1). During this quick-start strategy, the teacher moved around the room, addressed individual confusions, gave students an opportunity to rehearse their contribution, and prompted students with questioning.

***Thematic category 3: Questioning.*** Whether in whole group, small group, or individually, the teacher used questioning techniques to initiate student thinking, clarify thinking, enforce precision, and justify solutions and strategies. Sixth grade began a unit on the distributive property by working in groups to observe patterns in area models (SMP7 – Look for and make use of structure). Following a report out of observations and conjectures, the teacher introduced a definition of the distributive property in which he used the term 'expression.' He led the class in a whole group discussion to determine the meaning of expression:

What is an expression? Gus. [student responded] Okay, and that's something we often use kind of mixed with it, right - expressions and equations. Is there a difference between those? Yeah? [student responded] Zeb is saying, 'Wouldn't the equation have the answer?' So how could we differentiate? Irene? [student responded] Ok. So you're saying

you're thinking about order of operations with PEMDAS - you would use that with an expression. So how can we differentiate between an expression and equation?

The students had background knowledge that they were able to apply and the teacher used questioning to draw out prior knowledge and then help students draw a distinction. By activating prior knowledge, the teacher worked to help students make sense of problems and persevere in solving them (SMP1).

Students' responses revealed greater familiarity with equations, so the teacher used strategic questions to connect expressions and equations, "So an equation is just when you take an expression and you set it equal to something? . . . What could it be set equal to?" Next, he challenged students' thinking by suggesting that an equation could be created by setting an expression equal to another expression; "or another expression' - why did I put that last part in there? What sense does that make?" Finally, he asked students to test the claim by creating examples, "Can anyone give me a factor string, or two different factor strings, that equal the same thing?"

As students in the seventh grade class were preparing to work with angles along two parallel lines cut by a transversal, the teacher planned an instructional activity that built upon prior knowledge of measuring and labeling angles on a protractor (SMP1 – Make sense of problems and persevere in solving them). As he planned for the lesson, he anticipated students' struggles, "I think they're going to mix up the vocab. They have the vocab down in their notes but sometimes they don't go to their notes. So that's something I will have to prompt them to do - 'Go to your notes.'" After an exploratory activity in which students identified angles, the teacher led a review of the vocabulary terms by asking questions that required students to clarify thinking and provide justifications:

So somebody give me some adjacent angles. Rachel. [Student responded] Angles 6 and 8. Someone explain to me if they believe Rachel is correct with six and eight being adjacent and why. Is Rachel correct, Robbie? Why? [Student responded] Because they're right next to each other. Now if I were to say, 'Why does that make them adjacent?' Based on a definition, what would you say? Michelle. [Student responded] Ok. They have a common vertex and a common side and that's what makes them adjacent.

The teacher's questions provided an opportunity for students to express mathematical precision (SMP6 – Attend to precision). By asking students to judge the reasonableness of a proposed solution, the teacher supported students in constructing viable arguments and critiquing the reasoning of others (SMP3).

## **Summary**

The purpose of this qualitative case study was to identify discourse-based structures and practices used by a middle school teacher to support student engagement with Standards for Mathematical Practice (SMPs). The case study was conducted in the classroom of a middle school mathematics teacher in rural Maine. The teacher provided mathematics instruction for students in grades six, seven, and eight. Data were collected using classroom observations, interviews with the participating teacher, and review of curriculum documents used during observed lessons. Data analysis occurred in an iterative cycle throughout the observation period as characteristics emerged. Codes were created, indexed, and sorted to identify themes related to each research question.

For the first research question, two major themes emerged with respect to expectations for the ways students engage in mathematical discourse. First, the teacher worked to bring awareness of students' own thinking processes into the discussion. Discourse episodes reflected

the teacher's focus on the variety of approaches students used to confront mathematical problems. Second, during small group and whole group discussions, students were expected to use materials, especially their notes, as a resource to support their thinking. The teacher referred students to previous examples and definitions to justify their strategy or when working with peers to apply rules and procedures to problems in new situations.

For the second research question, multiple examples of all eight SMPs were identified throughout the 19 class periods observed. Discourse episodes revealed greatest emphasis on SMP3 (Construct viable arguments and critique the reasoning of others) with more than twice as many incidents identified than any other single SMP. SMP1 (Make sense of problems and persevere in solving them) and SMP7 (Look for and make use of structure) also had high frequencies of occurrences. Across all SMPs, oral expression of thinking, perseverance, precision, and group consensus were stressed.

For the third research questions, small group discussions, time for independent thinking, and teacher questioning emerged as the three most common teacher-centered instructional strategies used to engage students with discourse around SMPs. While teacher questioning was used to focus small group discussions and independent thinking time, in these situations, the questions were used as a guide to help students access the content. Questioning was connected to every SMP except SMP5 (Use appropriate tools strategically). Small group discussions were employed as students worked together to engage with SMPs within the context of mathematical problems. Independent thinking time supported a variety of SMPs but was the dominant strategy used in connection with SMP1 (Make sense of problems and persevere in solving them).

The unit of analysis in this case study was the teacher's instructional practices implemented to engage students with discourse related to SMPs. Although data analysis reveals

redundancies among themes related to expectations, instructional strategies, and teacher emphasis of SMPs, the lack of cultural diversity within the student population may result in conclusions that do not hold in student populations with greater diversity. Based on the evidence presented in this chapter, the next chapter will discuss conclusions and recommendations about discourse-based instructional strategies related to SMPs.

## **Chapter 5: Discussion and Conclusion**

### **Introduction**

Studies of national and international mathematics assessment results have shown that student achievement declines from elementary school through middle school and continues into high school (Lewis, 2013; Nation's Report Card, 2015; NCTM, 2014). During the same time period, student engagement in mathematics also declines as measured by decreased participation, more negative attitude, greater anxiety, and less confidence (Hannula, 2012; Way et al., 2015; OECD, 2014). While a positive correlation between mathematics engagement and achievement exists, no causal relationship has been definitively determined. Bobis, Anderson, Martin, and Way (2011) noted the potential for discourse-rich instructional practices to promote engagement and motivation in middle school mathematics courses. Large-scale studies have investigated classroom discourse and other factors influencing student engagement (Attard, 2013; Darragh, 2013) and noted in their conclusions that additional research was needed to understand the teacher's role in enacting practices that promote student engagement.

The 2010 release of the Common Core State Standards established a common foundation for state level academic standards in mathematics and English language arts. Along with grade level content standards, the eight Standards for Mathematical Practice (SMPs) outlined in the Common Core State Standards identify key skills, processes, and habits “that mathematics educators at all levels should seek to develop in their students” (p. 6). These proficiencies describe ways that mathematicians think and behave as they engage with mathematical concepts. Based on the importance of SMPs in mathematics instruction and the potential for discourse-rich instruction to improve student engagement, this qualitative case study was conceived. The purpose of the study was to identify discourse-based structures and practices used by a middle



school mathematics teacher to support student engagement with the eight Standards for Mathematical Practice (SMPs). This study was guided by the following research questions:

1. How were norms, routines, and classrooms expectations established and reinforced to support student discourse?
2. How were Standards for Mathematics Practice emphasized during instruction?
3. How were teacher-centered instructional strategies implemented to engage students with discourse around SMPs?

Since the SMPs describe the ways that students “increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise” (NGO & CCSSO, 2010, p. 8), the content of the mathematical engagement in this study was defined in terms of the SMPs. The process under investigation in this qualitative case study was the discourse-based strategies used to engage students with the SMPs. The focus of this study was on instructional practices enacted by a middle school mathematics teacher, not student responses to instruction. As such, data collection in this qualitative case study related to the teacher and his pedagogical choices.

This chapter contains a detailed discussion of the findings of the study in light of existing and known literature about discourse based instructional strategies and student engagement with mathematical practices. The discussion focuses on the contributions of the findings in light of current literature in the academic field. Further, this chapter contains the conclusion of the study and how these conclusions could influence the professional practice of middle school mathematics teachers seeking to improve student engagement with SMPs. Limitations of the study are also presented, along with practical and future implications. Recommendations for future research, as well as for the effective practice of teaching middle school mathematics, conclude this chapter.

## Summary of the Study

Previous studies have shown that discursive practices are characteristic of highly engaged mathematics classrooms (Edwards & Townsend, 2012; Darragh, 2013). Effective discourse practices in the middle school classroom have the potential to support the development of positive mathematical identities (Anderson, 2010; Boaler, 2013) and are directly connected to mathematical motivation (Martin, Anderson, Bobis, Way, & Vellar, 2012). Teachers and their instructional practices have the potential to impact, both positively and negatively, students' willingness to view themselves as being capable of doing mathematics (Boaler, 2013; Coomes & Lee, 2017; Yackel & Cobb, 1996). At the same time, the Common Core SMPs explicitly seek to identify practices that reinforce positive mathematical identities. In fact, NCTM (2014) identified discourse-based interactions as an essential feature of mathematics instruction, stating, "effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas" (p. 29).

The goal of this study was to identify discourse-based structures and practices used by a middle school mathematics teacher to support student engagement with SMPs. The identification of such structures and practices, or lack of, provides important information for teachers and those who train and support them. The information generated from the results of this study provide suggestions for middle school mathematics teachers and administrators as they work toward developing in their students the types of expertise characteristic of proficient mathematicians.

In order to understand how discourse was used as an instructional strategy to engage middle school students with Standards for Mathematical Practice, 20 classroom observations and five interviews were conducted at a rural middle school in Central Maine. Copies of student materials were collected for each lesson observed. An enactivist approach was employed, which

allowed the teacher to actively participate in an ongoing process of lesson reflection and analysis of transcripts from classroom observations and previous interviews. Brown and Coles (2012) argued that through an enactivist approach, teachers were able to observe patterns over time that brought awareness of processes and practices impacting student learning. As such, the enactivist approach results in the co-emergence of theories. Involving the teacher in the process of analyzing his instructional choices and their impact was an important consideration as studies suggest that reliance on evidence of student learning and responsive use of data to adjust instruction are characteristics of high-achieving schools (Hargreaves and Shirley, 2012).

Data analysis occurred in an iterative process that started early in the observation period. As instructional strategies were observed and identified, the teacher was encouraged to identify his thought process in relation to the learning intention and to evaluate the effectiveness of the strategy he used. To support the teacher's goal of continued improvement, discussions between the teacher and the researcher also provided an opportunity for the teacher to reflect upon changes he might implement if he were to repeat this lesson again.

Throughout the research process, incidents of SMPs were noted with open codes in a spreadsheet and the dominant instructional activity was identified. As new information became available in subsequent observations and interviews, columns were added to the spreadsheet to track open codes for teacher choices related to each of the SMPs. Themes for each of the research questions were identified through a process of axial coding as trends emerged. These themes will be presented in this chapter within the context of known literature about discourse-based mathematics instruction. Contributions of these findings are articulated and recommendations made that could improve the use of discourse as an instructional strategy to increase student engagement with the SMPs.

## Summary of Findings and Conclusions

The focus of this study was on the teacher and his actions to support student use of discourse as they engaged with SMPs. Prior research provided insights into the benefits of engaging students in discourse throughout the learning process. Student discourse provides a quick and easy way for teachers to engage in formative assessment and immediately implement targeted intervention practices (Coomes & Lee, 2007). Student motivation and engagement have been shown to improve with increased discursive activity (Way, Reece, Bobis, Anderson, & Martin, 2015). Studies in mathematics and other content areas offer suggestions for effectively implementing discourse with middle school students as a component of the learning process (Fisher, Frey, & Rothenberg, 2015; Nichols, 2006; Michaels, Shouse, & Schweingruber, 2007). However, there is little research available that specifically investigates the ways that discourse-based instructional strategies are used to engage middle school students with SMPs. To address this gap in research, information was collected in the current study to illuminate such practice. Results of the study are presented in this section to address the research gap.

**Norms, routines, and expectations for discourse.** Question one was devised to identify classroom norms, routines, and expectations that supported the use of discourse. Research highlights the importance of establishing routines and norms to create a safe learning environment where students are willing to engage in complex tasks (e.g., Buchheister, Jackson, and Taylor, 2015; Stephan, 2014). Although research provides guidance about the types of norms that might be useful for promoting student discourse in a mathematics classroom (Webb et al., 2014), traditional norms for group interactions were absent from the classroom observed in this study. There were no protocols in place to guide student engagement, to focus students' ideas on mathematics contents, nor to support productive interactions when conflict arose.

From the perspective of establishing and supporting effective mathematical interactions, the absence of norms for student discourse was problematic. Independence and effort were terms used often by the teacher as he described how students should be interacting with new material and their peers without relying on the teacher as the authority. Without structures for how to engage with one another, students are conditioned to view the teacher as the only authority capable of providing explanations (Yackel & Cobb, 1996). Classroom norms, properly instituted can shift this view. Routines that establish roles for student engagement help to shift the teacher's role "from 'being *the* authority' to 'being *in* authority'" (Reznitskaya et al., 2012, p. 288). Yet, no models were provided for students to follow nor resources provided for how to proceed when problems were encountered. The lack of specific routines or expectations when working together in groups created frustrations for the teacher that he discussed during several interviews. Despite the conspicuous absence of formal norms and routines for student discussions, data analysis reveals that two common themes emerged in relationship to expectations for student engagement with SMPs.

***Thematic category 1: Discourse as a means of practicing metacognition.*** Based on the data presented, the teacher established expectations for students to recognize and actively monitor their metacognitive processes as they engaged with mathematics content. Awareness of cognitive functioning is viewed as "fundamental for generating well-reasoned arguments, as it permits ongoing evaluation of both processes and products of thinking" (Reznitskaya et al., 2012, p. 289). Within the first week of school, the teacher introduced exploratory activities that offered multiple solution strategies and asked students to intentionally focus on the patterns their brains noticed. The use of such activities early in the year implied that different patterns existed and there was not one single correct way to approach problems, thus providing multiple access points

and supporting perseverance (Lewis & Ozgun-Koca, 2016; Wachira, Pourdavood, & Skitzki, 2013).

Even with the focus on metacognitive practices, the teacher established himself as the mediator of information. He initiated the sharing of strategies through a whole-group debrief and he called on students (granting permission) to share their strategies and the patterns they noticed. Lewis and Ozgun-Koca (2016) suggested that open-ended activities, such as the one implemented in the lesson described, create opportunities for students to think and process. The teacher's role is to create and maintain the space to engage in metacognitive practices. The focus on metacognition supports perseverance (SMP1), student argumentation (SMP3) and recognition of patterns (SMP7). Shared experiences of metacognitive awareness have the potential to further emphasize these SMPs by engaging students at the edge of a collaborative zone of proximal development; helping students "monitor the degree to which they understand each other's thinking, extend other's ideas and apply them in new ways, acknowledge divergent interpretations, and resolve inconsistencies between ideas proposed" (Webb et al., 2014, p. 80). However, the process for sharing did not require students to consider the experiences shared by others.

***Thematic category 2: Notes as a resource for discussion.*** The second expectations established by the teacher to support student discourse was the use of notes to initiate and sustain student discussions and engagement with content. To support his explicit emphasis on independence, the teacher expected students to use resources, especially notes from lessons and previous assignments. Although note-taking appeared to be a solitary activity, the notes, once recorded, were expected to be used as a resource to support students talking with one another. During group discussions, the teacher moved between groups, attuned to common confusions

and misconceptions. As he noticed that several groups were struggling with the same concept, he reminded the class to go back and use their notes. NCTM (2014) suggested a strategy for situations similar to this whereby, noticing a common struggle, the teacher would call together a discussion on the topic. The teacher, with knowledge about the developmental progression of skills and knowledge and familiarity with common misconceptions, should assume the role of discussion facilitator, asking students to explain their confusions. Without doing the cognitive work for students, he can support them through productive struggle by helping them work together to find similar problems (Williams-Candek, 2016). Simply telling students to use their notes may not have been a useful strategy for students who did not take notes or whose notes were not complete. Further, this strategy is ineffective for supporting discussions as it is possible that students could access their notes and then move on without engaging with others.

During an interview late in the data collection process, the teacher stated that he specifically sought to remove himself from the head of the class, instead attempting to create opportunities for students to talk and explore concepts together using their notes and peers as a resource. During which time, he hoped to be able to move between groups to monitor progress and become a facilitator for student learning. This was a change he struggled to maintain, as students were reluctant to accept the mathematical expertise of their peers. Although the teacher consistently provided opportunities for students to discuss mathematical concepts together, explicit routines for group interactions (how to work through disagreements, steps to try before asking the teacher, etc.) were not evident and through multiple interviews the teacher acknowledged this as an area in which he hoped to improve.

**Emphasis of SMPs during instruction.** Research question two reflected the centrality of the SMPs to mathematics instruction as outlined in the Common Core State Standards for

Mathematics (2010). As noted, the eight SMPs outline desirable skills, processes, and habits that mathematics teachers should teach and reinforce in conjunction with content knowledge.

Throughout the data collection period, SMPs were noted and charted. Out of 139 observed incidents related to the eight SMPs, 97 incidents were related to emphasis of three SMPs. In order of highest frequency, SMP3 Construct viable arguments and critique the reasoning of others, SMP1 Make sense of problems and persevere in solving them, and SMP7 Look for and make use of structure, were observed more than twice as often as the other five SMPs combined. The expectations for SMP3 are directly connected to the use of discourse but also explicate that student discussions employ higher order thinking and reasoning.

It is important to note that the eight SMPs are not mutually exclusive. SMP1 states in part, “Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends” (NGA & CCSSO, 2010, p. 6). To effectively demonstrate the expectations identified in SMP1 students must simultaneously make use of structure (SMP7) and communicate about their understanding (SMP3). The high frequency of occurrences of these three SMPs aligned with the expectation for self-awareness of mathematical thinking (metacognition) and independence (as reflected by emphasis on independent use of notes) observed in this study. As should be expected, these same SMPs featured prominently in connection to the dominant instructional strategies used by the teacher.

More important than which SMPs were used, analysis of the SMPs sought to determine how the SMPs were emphasized during instruction. Across the 139 observed incidents, four themes were identified. First, the teacher expected students to *persevere* as they worked to make sense, reread, and try different strategies. As part of his regular instructional repertoire, the



teacher asked students to *express their thinking* as they explained processes, justified their solutions, and judged the reasonableness of others' responses. He also expected students *to use precision* when communicating with mathematical symbols, terminology, and when representing quantities. Finally, through group activities, he frequently required students to *reach consensus* as they applied criteria and rules, analyzed nuances of precise mathematical language, and developed solutions that demonstrated conceptual awareness.

***Thematic category 1: Perseverance.*** The first thematic category reflects a key productive disposition impacting student engagement. As such, it should not be surprising that SMP1 outlines expectations for students to 'Make sense of problems and persevere in solving them.' Self-regulatory behaviors, such as perseverance, "have been found to be conducive to both motivation and achievement" (Way et al., 2015, p. 632). In a 2013 study, Wachira et al. concluded that it is possible for students' mathematical dispositions to be altered over time by pedagogical practices implemented by the teacher.

In the current study, perseverance was emphasized through instructional activities that transferred cognitive responsibility to students. One characteristic of perseverance is that students are able to consider analogous problems when they encounter struggles. When eighth grade students struggled to make sense of how they might square an imperfect square ( $\sqrt{3} \times \sqrt{3}$ ), the teacher asked them to work in groups as they considered similar problems with perfect squares to see if they could determine a rule. This sequence of learning required students to perform mathematical computations, but it also required that they attend to the processes and patterns that developed (SMP7 – Look for and make use of structure). Instructional expectations that focus on perseverance reinforce the idea that "learning is a process determined by effort" (Boaler, 2013, p. 145).

Teachers support students' perseverance when they "find ways to support students without removing all the challenges in a task" (NCTM, 2014, p. 49). As problems became increasingly complex, sometimes a simple acknowledgement by the teacher that something was different was the only prompting students needed to engage with the content. When solving for missing sides of a triangle using the Pythagorean Theorem, the teacher asked students to begin by considering a problem where the missing value was a leg (previous problems had asked students to calculate the length of the unknown hypotenuse). To begin, he asked them to work as pairs to compare the diagrams for the new problems with diagrams from previous problems and only identify what was different. Comparing the diagram and the equation supported students with modeling (SMP4) and gave them an opportunity to decontextualize as they considered variables in an equation absent their quantitative referents (SMP2 – Reason abstractly and quantitatively). Small groups shared their ideas as they tested their observations, a process that supported the teacher's intention of shifting ownership for learning to students.

Opportunities for students to demonstrate perseverance were evident in the teacher's practice and observed in relation to each of the research questions. Classroom expectations supported the teacher's emphasis on perseverance. The teacher engaged students in discussions about what they recognized as familiar in new mathematical problems, supporting metacognition (Felton, Anhalt, & Cortez, 2015; Wilburne et al., 2014). He encouraged students to use their resources to collaboratively search for answer to their questions. Providing students with opportunities to engage in discourse-based strategies that helped them persevere in challenging situations aligned with research supporting student engagement with mathematics content (Warshauer, 2015).

***Thematic category 2: Express thinking.*** SMP3 (Construct viable arguments and critique the reasoning of others) is the SMP most closely tied to classroom discourse. As such, it is not surprising that this practice was identified more frequently than any other practice throughout the observation period. Not only was the expectation for arguments and criticism evident in relation to SMP3, but the expectation for students to express their thinking orally was also evident in relation to all of the practices.

The discussions that occurred throughout a sixth grade unit on ratios highlighted the interconnectedness of verbal expressions with multiple SMPs. As students constructed models (SMP4) for their ratios, they were expected to communicate about the validity of the model (SMP3), connecting the abstract model to the quantities being compared (SMP2). As the unit progressed, students were asked to explain how their model could be scaled through repeated iterations to represent increasingly larger quantities (SMP8).

The teacher shared his belief that student understanding was greater and engagement increased when he yielded the floor for students to engage with one another. Herheim (2015) and Lewis and Ozgun-Koca (2016) suggested that the teacher should take seriously his role to provide the time and space for students to think through complex problems. By allowing time and space for students to collaboratively create models and consider their application in context, students not only made sense (SMP1) but they had access to multiple points of entry. Due to the text-laden format of the problems, the teacher anticipated that some students may initially resist engaging with the problem and others might struggle to persist. The teacher monitored and supported student progress by asking students to explain their models.

The teacher's knowledge of mathematical concepts helped him to identify misconceptions in student responses. The eighth grade lesson on exponents provided evidence

that the teacher recognized a misconception about an unwritten exponent of one being mistaken as no exponent. He provided examples and orchestrated a discussion that led students to clarification of their understandings related to this important algebraic concept. This interaction highlighted the importance of the teacher in recognizing and supporting the evolution of student understanding (Wachira, Pourdavood, & Skitzki, 2013).

As the teacher asked students to share their ideas, he often asked other students to judge the correctness or completeness of the response. This sequence was often followed up with a request for an explanation (SMP3 – Construct viable arguments and Critique the reasoning of others). The term ‘mistake’ was used by the teacher 22 times over 20 observations. Often the teacher invoked this term when he wanted to call attention to a common misconception or a common procedural error. Students also interacted with common errors through incorrect worked examples. As students reviewed incorrect worked examples, they worked in pairs and analyzed procedures others had used. In debriefing worked examples, the teacher asked students to not only attend to precision (SMP6), but to explain how such confusions might occur. Worked examples have been associated with greater metacognitive awareness, improved procedural precision, and conceptual fluency (Boaler, 2013; McGinn, Lange, & Booth, 2015).

***Thematic category 3: Use precision.*** Precision refers to those qualities that aid clear and effective mathematical communication. Under Maine law, current eighth grade students will need to demonstrate proficiency as a ‘clear and effective communicator’ in order to earn a high school diploma (Maine Revised Statutes, 2011). As such, it was not surprising that precision emerged as an emphasis across SMPs. In addition to computational precision, clear and effective mathematical communication is characterized by accurate use of terminology, symbols, and unit labels (NGA & CCSSO, 2010). Although the expectation to attend to precision is its own SMP

(SMP6), emphasis on precision, especially related to terminology, use of symbols, and labeling, was noted across multiple SMPs in the study.

Whether guiding students in a unit on geometric angles or a unit on ratios, the teacher repeatedly prompted students to organize and monitor their models by labeling the referents. As students shared their conjectures about the concepts (SMP3), the teacher asked them to clarify their statements by referring to the model (SMP4) and identifying the quantities being represented (SMP2). His emphasis on precision was less about communicating the correct answer than it was about explaining clearly the process used to arrive at the answer. As students worked to express their thinking, they were expected to use precise language. While solving a multi-step problem in which students had to isolate a variable, they were prompted to use terms such as ‘inverse’ and ‘reciprocal’ in their explanations (SMP3). The techniques used by the teacher to emphasize precision aligned with the NCTM (2014) recommendations for teachers to implement instructional strategies that require students to make choices about methods and strategies, to expect students to explain their approaches, and to produce defensible solutions.

***Thematic category 4: Reach consensus.*** The expectation for students to reach consensus was identified most often in relation to student explanations (SMP3). The teacher framed consensus in terms of agreement. As students worked together in groups, they were expected to come to agreement on a solution or a common explanation. This emphasis on agreement was observed as students worked together to determine whether or not a series of graphs represented functions (SMP7), as students sorted shapes based on attributes (SMP8), and as students worked to solve a problem and present their solution on the board (SMP3). In an interview, the teacher was asked to share his intention for having students work together to reach consensus. He expressed that he hoped students would consider one another’s solutions, further supporting his

effort to build students' confidence in their ability to generate mathematical knowledge and to remove himself as the ultimate mathematical authority. When they did not readily reach consensus, he hoped that they would engage in a conversation with their group members to seek clarification. He was disappointed that students were instead willing to yield and assume the response of one student – presumably a student whom the group identified as mathematically superior (Aguirre, Mayfield-Ingram, & Martin, 2013). In this case, protocols for sharing solutions and explicitly engaging in reasoned defense might have supported the types of interactions the teacher desired.

Consensus, in the way it was communicated by the teacher in this study, stifled discourse (Franke et al, 2015). When the goal was agreement, the conversation ended as soon as the group was able to determine which solution or strategy to adopt. The teacher did not provide sentence starters for how students might ask questions to understand their classmate's solution (SMP3). There were no expectations to consider multiple strategies before arriving at a decision. There were no protocols for ensuring that each member of the group was understood (SMP1) and was able to articulate the steps used to arrive at the group's solution. Any of these strategies suggested by Webb et al. (2013) might have provided the support needed to invoke students as able generators of mathematical knowledge. These strategies support Leinwand's (2009) assertion that "good mathematical thinking *begins* with an answer" (p. 71).

**Instructional strategies for discourse with SMPs.** The purpose of question three was to illuminate the specific actions and decisions implemented by the teacher to support student engagement with SMPs through the use of discourse. Walshaw (2014) stated that teaching strategies that aim to mediate the effects of unequal access to knowledge and resources are characterized by "co-participating as a learner in a community of learners" (p. 3). Data analysis

for question three focused on instructional strategies used by the teacher to engage students with discourse related to SMPs.

Three general themes emerged related to instructional strategies used to engage students with discourse around SMPs. *Small group discussions* were often used as a format to engage more students in discussions related to mathematical concepts and procedures. As the observation period progressed, the teacher made more frequent use of *opportunities for independent thinking* prior to having students discuss together. The teacher also relied on *questioning* techniques individually, in small groups, and with the whole class to prompt student engagement with SMPs.

***Thematic category 1: Small group discussions.*** The most dominant strategy observed in this study to promote student discourse was small-group discussions. Discussions ranged from brief and specific in nature to longer and more exploratory. Students were frequently asked to turn quickly to a neighbor to develop a definition based on an observed pattern, such as determining the rule for communicating with exponents. These brief episodes of turn-and-talk supported student sense-making (SMP1) and prompted students to think about the patterns they were able to identify (SMP7).

In longer, more exploratory episodes, students worked together to build conceptual knowledge of relationships leading to the Pythagorean Theorem and to design models (SMP4) for rates and ratios. In these more protracted interactions, students were given sets of instructions to follow – create a table of lengths that work to form a right triangle, develop a tape diagram to model the relationship between quantities. Students worked together in groups to complete activities, but objectives for the lessons were often presented as the completion of an activity.

Webb et al. (2015) noted that empirical findings support the benefits of group participation for student learning. While students did appear to be actively engaged during group activities observed through the current study, or at least compliant, it is important to note that distinctions have been drawn between behavioral (or operative) engagement, cognitive engagement, and emotional (or affective) engagement (Attard, 2013; Hannula, 2012; Way et al., 2015). Research conducted by Attard (2015) further suggested that student engagement with mathematics, across each of the domains of engagement, was highly dependent upon strong teacher pedagogy. While group participation is commonly associated with high student engagement (e.g., Stephan, 2014; Brooks & Dixon, 2013; Walshaw & Anthony, 2008), strong expectations are needed to support effective interactions within the group setting (Stephan, 2014; Felton, Anhalt, & Cortez, 2015). Small group discussions provide opportunities for students to engage with SMPs (recognizing patterns, making sense, modeling, etc.). Teachers must consider the purpose for discussions and activities. Task-completion objectives may be more efficiently realized through individual student activities. But if the objective is to engage with mathematic concepts from multiple perspectives, group activities may be better suited for the purpose.

***Thematic category 2: Opportunities for independent thinking.*** To support self-awareness of how students were thinking about information (making sense of problems – SMP1) and potential strategies for solutions, the teacher integrated time into instructional activities in which students were expected to think and plan prior to engaging with others. Individual time to think and process information initially emerged as a strategy to promote individual accountability; the teacher worried that there might be an imbalance of cognitive engagement as some students yielded to their peers whom they presumably viewed as more mathematically capable or more out-spoken.



The teacher attempted to engage more students in class activities and discussion by providing individual time to think and plan in advance of group work or full class discussions. After a brief time to consider a problem or identify a strategy for organizing data, students were asked to share their initial thinking, either in small groups or in a whole group setting, as they worked to make sense of problems (SMP1). While evidence suggests that breaking the lesson into discrete chunks of individual, small-group, and whole-group activities might “slow down the problem-solving process and encourage perseverance by staving off the rush to obtain a solution” (Lewis & Ozgun-Koca, 2016, p. 110), the teacher in the study had few mechanisms in place to support and monitor the effectiveness of this strategy.

A classroom poster, a list of questions for students to keep in their notebooks, or a few prompting questions displayed on the board may have helped some students who were unsure how to think about a topic in which they felt uncertain. The use of an exit ticket or learning log may have also provided a forum for students to capture and monitor their struggles and the methods they used to overcome such stumbling blocks. Prompts to encourage reflection on unfamiliar vocabulary terms or to begin creating and labeling a model can give students a place to initiate their thinking and also serve as a valuable piece of formative assessment to the teachers (Coomes & Lee, 2017).

***Thematic category 3: Questioning.*** During whole group lessons, the teacher used questioning to initiate student thinking (SMP1), often asking, ‘What do you recognize? How is this problem like one we already did?’ The use of *focusing questions* like these helped the teacher attend “to what the students are thinking, pressing them to communicate their thoughts clearly” (NCTM, 2014, p. 37), thus promoting both metacognition and discourse. Questions that elicited recognition of familiar patterns (SMP8) were common at the beginning of a new unit of

instruction to stimulate student thinking. At advanced stages within a unit, questioning strategies were also used as a way to transition into more complex applications. The teacher's use of questioning strategies not only engaged students in discourse about mathematical concepts, but also prompted their engagement of SMPs, notably 'Look for and make use of structure' (SMP7), 'Make sense of problems and persevere in solving them' (SMP1), and 'Construct viable arguments and critique the reasoning of others' (SMP3).

As the content expert, the teacher used questioning to move students along a learning trajectory toward the grade level expectation. He invoked prior understanding by introducing concepts in small chunks that represented expectations of proficiency with prior grade level standards. In so doing, he monitored student understanding and identified students who may need one-on-one attention or reteaching. The use of questioning to monitor student progress and provide feedback has been identified as a high-impact instructional strategy, often linked to formative assessment (Chapuis, 2014; Heritage, 2016). The use of guiding questions, open-ended questions, and student-initiated questioning highlight the shifts from teachers as moderator of knowledge and knowing to students as initiators and facilitators of their own learning (Leinwand, 2009).

## **Implications**

To address the gap in literature, the research was conducted to examine teacher actions related to the implementation of discourse-based instruction. This qualitative case study was designed to identify discourse based structures and practices used by a middle school mathematics teacher to support student engagement with SMPs. The following sections discuss theoretical, practical, and future implications for scholars and practitioners. Strengths, weaknesses, and credibility of the study are also addressed.

**Theoretical implications.** This study draws upon the long-established role of discourse to support learning. Socrates, Plato, and, more recently, Dewey, believed that through discursive interactions, participants built upon their prior knowledge and interests as a guide to new learning and sustained intellectual curiosity (Fisher, 2013; Nystrand, 1997). Piaget, Vygotsky, and Bahktin expanded on the connection between discourse and learning processes. Vygotsky and Piaget each emphasized the importance of discourse to stimulate personal knowledge development (Kazak, Wegerif, & Fujita, 2015). As students confront new information, generally new information creates disequilibrium with prior knowledge; learning occurs because the new information is internalized and reshapes prior understandings. In the current study, the teacher's emphasis on metacognition aligns with the belief that learning is an individual cognitive activity.

Bahktin agreed that learning occurs as the result of tension between prior knowledge and new, developing understanding. Bahktin viewed discourse as the vehicle through which new information could be introduced, thus initiating the learning process (Kazak, Wegerif, & Fujita, 2015). As such, learning was seen as the result of an external process contingent upon conversation. The use of small group and whole group conversations evident in the current study align with Bahktin's view of learning as an external social process.

Important in the theories proposed by Piaget, Vygotsky, and Bahktin is the belief that interactions guided by a 'more knowledgeable other' facilitate the process of meaning making within a 'community of inquiry' (Fisher, 2013; Reznitskaya et al., 2012). Therefore, the role of the teacher is important to the process of creating and managing learning interactions. The current study offers evidence that the teacher understood the significance of his role as he facilitated conversations and frequently asked students to consider the explanations presented by their peers. However, it must be noted that the line of questioning observed throughout the study

rarely asked students to consider how new information either supported or challenged their current understanding. Discursive learning theories suggest that student learning may be improved if students are intentionally provided with opportunities and strategies with which to confront cognitive dissonance (Kazak, Wegerif, & Fujita, 2015). Metacognitive awareness is a step in that direction, but the practices observed in this study stop short of providing evidence that cognitive dissonance was provoked.

**Practical implications.** This qualitative case study addresses discourse based instructional strategies to improve student engagement with mathematical practices identified in the Common Core State Standards (NGA & CCSSO, 2010). Previous studies have indicated a positive correlation between student engagement and achievement in mathematics (Hannula, 2012; Way et al., 2015). Based on the results of this study, instructional strategies are implicated that could improve student engagement in middle school mathematics. By better understanding how one teacher uses discourse-based structures to improve engagement with specific mathematical practices, teachers and those who support them, might benefit. Analysis of the data revealed redundancies between the results of the three separate research questions.

In this study, the teacher worked to engage students with mathematical content and SMPs through the use of discursive practices that emphasized perseverance, precision, consensus, and verbal explanations. The implementation of norms that emphasized metacognitive reflection is echoed in the expectations for perseverance and the use of verbal expressions to express thinking. Also, norms for student use of notes supported expectations across all four themes connected to the SMPs – perseverance, precision, consensus, and expressing thinking. Through repeated emphasis on key skills and dispositions for mathematical efficiency, and norms to support such

outcomes, teachers are able to consistently communicate expectations for student engagement with content. Such redundancies should be developed in middle school mathematics classrooms.

Likewise, analysis of the instructional strategies implemented throughout the study indicated redundancies with the norms identified and the themes that emerged within the SMPs. Within small group discussions, students were expected to use their notes as a resource for precision, expressing thinking, and reaching consensus. Independent thinking time was presented as a means to practice metacognition, persevere, and to begin formulating thoughts to be shared during group work. The teacher used questioning strategies to help students persevere, to reflect on their understanding, and to support students' precise mathematical communication. The instructional strategies identified in this study support the role suggested by the theoretical literature of the teacher as a 'more knowledgeable' guide for student learning. With an understanding of learning as a process of engaging with new material and increasingly complex processes, teachers can create learning opportunities that guide and support students.

Discourse is an easily accessible instructional strategy. Middle school mathematics instruction should seek to expand implementation of discourse-based practices as a means to engage students with increasingly complex mathematical concepts. Norms and routines have been shown to provide support for effective student interactions during learning (Buchheister, Jackson, & Taylor, 2015). In support of perseverance, individual think time should be implemented prior to group engagement. Learning experiences that help students develop "metacognitive awareness of themselves as learners, thinkers, and problem solvers" are suggested as key characteristics of effective mathematics teaching (NCTM, 2014, p. 9).

**Future implications.** One limitation of this study was the lack of diversity within the student population. The lack of diversity meant that the teacher was not required to consider nor

plan for variations of communication patterns often observed in more diverse populations (Aguirre, Mayfield-Ingram, & Martin, 2013; Walshaw, 2014). In more diverse groups, suggestions for norms and instructional activities specific to discursive interactions could be beneficial. Although the lack of diversity could not have been adequately addressed through a larger local sampling, future studies in more diverse areas could provide useful data.

Future studies situated within the limited demographic setting of the current study could be strengthened by implementing specific instructional models designed to address SMPs. Additionally, inclusion of student data ranging from achievement data to student perception surveys or interviews could provide another perspective on the effectiveness of discourse as an instructional strategy in mathematics. The data from this study provides an interesting starting point for considering the intentions of the teacher and his perceptions of the impact of his instructional choices. Including additional data sources may provide additional insights about the realized impact of the strategies.

The research questions used to guide this study may also represent a limitation of this study. As a result of the initial literature review that guided the development of this study and the research questions, it was assumed that norms and routines for classroom discussions would be established. In the initial design proposal, the researcher planned to collect classroom documents in which expectations were conveyed to students. Indeed, multiple classroom observations were conducted in the first week, including three observations of first meetings with particular classes, in order to capture data about how expectations were established. Therefore, the researcher entered into this study with a pre-conceived notion about the types of norms that she expected to see and the way she expected to see them. The researcher attempted to identify her biases within the conclusions. During the study, member checking and data triangulation confirmed the

emergence of trends related to norms and expectations within the data. Future researchers should establish clear definitions of norms and their role within instruction.

**Strengths and weaknesses.** This study focused on one teacher's use of discourse as a means for engaging middle school students with the SMPs identified in the Common Core State Standards for Mathematics (2010). The results of the study are limited by the lack of diversity in the school where the research occurred. With a population of students that is 98% white and 100% English speaking, the teacher's practices were not significantly impacted by cultural nor language challenges. As such, the results may not be generalizable to more diverse populations of students. However, the lack of diversity within the sample population may also strengthen the overall conclusions of the study that discourse-based instructional practices can be implemented to support student engagement with SMPs. The redundancy of themes related to each of the research questions suggests that the instructional strategies, norms, and SMPs were aligned and supported an overall approach by the teacher to student engagement with mathematics.

The researcher's position as a district level administrator at the research site may also be considered a weakness of the study. Although the researcher had no evaluative capacity over the teacher, the teacher may have felt compelled to engage in practices outside of his usual instructional repertoire. To mediate this concern, classroom observations were unannounced. Signed letters were also collected from the principal and superintendent, assuring that the teacher's employment nor evaluation would be negatively affected by any data or results emerging from the research process.

The enactivist approach of this study helped to diminish the concern about the researcher's administrative position. Throughout the research process, the teacher was actively encouraged to analyze class transcripts and offer explanations of interactions. In this way, he also

developed plans for next steps as he worked to address misconceptions and plan for future learning activities. As such, the enactivist approach and the teacher's participation in the analysis process are seen as strengths and as means for enacting remedies to learning struggles.

## **Recommendations**

In this section, recommendations are provided to guide future research related to student engagement in mathematics classes. Additionally, key results are summarized for the practical application of discourse as a means of engaging students with SMPs. Finally, overall recommendations are provided for the practical application of findings from the current study.

**Recommendations for future research.** This study provides evidence to suggest that strategically planned discourse activities were planned to support student engagement with SMPs. The activities used and the resulting emphasis on SMPs revealed consistent themes throughout the study. The routines and norms for student discourse, although unconventional, appeared to support the instructional strategies used and themes within the SMPs. Although the findings support Leinwand's (2009) conclusion that, when it comes to planning for mathematics instruction, "punting is simply no longer acceptable" (p. 72), more specific research to support teacher practice would be helpful.

Maine middle school students and teachers have had the unique experience of having had access to technology in a one-to-one setting since 2002 (Maine DOE, 2015). In the school where this study occurred, every student in grades seven and eight had their own school-issued iPad Pro and sixth graders used MacBook laptops; the teacher also had both devices and the classroom was outfitted with an Apple TV, projector, and interactive whiteboard. Although documents were shared electronically between the teacher and students in the current study, future research should address ways to effectively leverage technology to support productive learning



dispositions in mathematics. For the past 15 years, Maine residents and lawmakers have made a significant commitment to providing 21<sup>st</sup> Century Learning tools to Maine students. Future research should be conducted to determine instructional strategies for the effective use of these tools to improve student engagement and achievement of mathematics?

The current study was designed to identify the instructional choices implemented by a teacher to engage students with SMPs. While the results of the study suggest dynamic interactions and alignment between SMPs, instructional strategies, and classroom expectations, this study should be expanded to determine the impact of these choices. In a 2012 study of the impact of dialogic instruction to promote transfer to new tasks, Reznitskaya et al. concluded that students in the treatment group (those who received instruction using discourse-based strategies) performed no better on new tasks than students in the control group. Additional research should be conducted to determine the transfer effect of discourse-based instruction that emphasizes SMPs to student achievement in mathematics. If the goal of improving student engagement is to impact student achievement, additional data collection could help to determine whether such a causal relationship exists, as Hannula (2012) suggested.

**Recommendations for future practice.** This study highlights the need for teachers to intentionally enact classroom practices that support discourse. The data presented in this study suggested that the teacher recognized the importance of establishing students as independent learners by creating opportunities for them to persevere with mathematical content and SMPs. The following recommendations for practitioners emerged from the conclusions of this study.

Middle school mathematics teachers should work to intentionally teach and reinforce protocols for student interactions. Expectations for interactions must move beyond agreeing or disagreeing. Expectations and protocols should be implemented that require students to justify

their reasoning about mathematical concepts. In the absence of protocols for how to negotiate beyond disagreement, students become reliant upon the teacher to resolve disputes, thus stifling independence and perseverance. When students lack clear roles during group interactions, more timid or insecure students may disengage or defer to others in the group, relinquishing responsibility for learning (Aguirre, Mayfield-Ingram, & Martin, 2013).

Teachers should plan specific lessons and learning activities to address SMPs in ways that support mathematical precision within the evolution of learning expectations outlined in the Common Core standards. A working familiarity with the standards and progression of skills within the standards is essential to making strategic choices about learning activities. In order to create opportunities for students to engage in appropriate levels of cognitive dissonance, an essential characteristic of discursive learning theories, teachers must know the developmental progression of skills and knowledge within the standards. Awareness of the evolution of the standards will help teachers to plan for learning opportunities that move students along a developmental continuum.

Once effective instructional practices have been designed, teachers should implement strategies to monitor the effectiveness of their instructional choices. As students work independently or in small groups, it becomes more difficult to monitor each student's progress. Assessment methods should be varied and designed to collect data on the important learning objectives. If thinking begins with the answer, as Leinwand (2009) suggested, assessments should move beyond reliance on a simple answer and, instead, require students to explain their thinking about the mathematical process used to arrive at the solution. Worked examples, problems with multiple solutions or solution paths, and exit tickets that elicit student reflections

on their thinking should all be considered as strategies for teachers to monitor not only students' thinking, but the effectiveness of their instructional practices.

Teachers and administrators who support them may benefit from this study as they consider the instructional environment and the nature of interactions within their mathematics classrooms. The theoretical frameworks anchoring this study suggest that learning activities should include opportunities for students to think independently and recognize their own level of understanding. Opportunities for group interactions are also considered important as they provide students with access to alternative views that move them to the edge of their zone of proximal development. Administrators should seek to support teacher practices that make use of flexible grouping practices and interactive learning environments. Providing access to professional development opportunities that promote strategies for effective interactions with SMPs is one way that administrators can support teachers' classroom practices.

The results of the current study support the centrality of the teacher in establishing practices and expectations that support student discourse as a means for engaging in mathematical practices. A prior study of the teacher's role in promoting classroom discourse concluded with a swift reminder that there is a delicate balance between encouraging student discovery and guiding a trajectory of learning toward grade level standards (Wachira, Pourdavood, & Skitzki, 2013). As the content expert, teachers cannot abdicate their responsibility to establish and shape learning opportunities through clear learning intentions aimed at grade level expectations.

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**Appendix A: Standards for Mathematical Practice (SMP) from the Common Core State  
Standards for Mathematics (2010)**

- 1 — Make Sense of Problems and Persevere in Solving Them
  - 2 — Reason Abstractly and Quantitatively
  - 3 — Construct Viable Arguments and Critique the Reasoning of Others
  - 4 — Model With Mathematics
  - 5 — Use Appropriate Tools Strategically
  - 6 — Attend to Precision
  - 7 — Look For and Make Use of Structure
  - 8 — Look For and Express Regularity in Repeated Reasoning
- (NGA & CCSSO-M, 2010)

## Appendix B: NCTM's (2014) Mathematical Teaching Practices

Mathematics Teaching Practices
<b>Establish mathematics goals to focus learning.</b> Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
<b>Implement tasks that promote reasoning and problem solving.</b> Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
<b>Use and connect mathematical representations.</b> Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
<b>Facilitate meaningful mathematical discourse.</b> Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
<b>Pose purposeful questions.</b> Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
<b>Build procedural fluency from conceptual understanding.</b> Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
<b>Support productive struggle in learning mathematics.</b> Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
<b>Elicit and use evidence of student thinking.</b> Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

(NCTM, 2014, p. 10)

## **Appendix C: Interview Protocol**

Interviews with the teacher will be conducted at least once every two weeks and recorded. Notes will be collected during the interview electronically. Additional clarification may be added to notes within the same day that the interview occurred. Interviews will be conducted in a semi-structured format, using the questions below as starters. Data collected from documents, previous interviews, and classroom observations occurring between interviews will be analyzed and used to focus future interviews.

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Date of Interview (at least once every 2 weeks):

Time:

Location:

Notes about setting (unusual activities within the school, changes to schedule, end of grading period, etc.):

Questions:

1. Let's talk about the lesson (either a past lesson or a future lesson).
  - a) What were (are) your learning targets for the lesson?
  - b) Which Standards for Mathematical Practice (SMPs) were (are) you hoping to have students use during this lesson?
  - c) What activities did (will) you specifically integrate into the lesson to engage students with the SMPs? (Follow up as needed to elicit an explanation of the qualities of the

instructional activities which were intended to engage student and any issues anticipated in advance.)

- d) What evidence did you (hope to) observe to indicate that students were engaging with the SMPs? (Follow up to determine if anything in the evidence was surprising.)
- e) Reflecting back, are there any changes you would make if you were to do this lesson again? Why?

2. Assuming that routines and expectations for discourse have been established and communicated to the class ...

- a) How did you create opportunities for students to participate in discourse during this activity?
- b) How did the level of discourse support the SMPs?
- c) What is (was) your role during this activity?
- d) Is the level of student discourse in this class adequate, or would you prefer to see changes? Explain. (If changes are desired, how might you facilitate that change?)

2. Analysis of specific discourse incidents ... (to be used as appropriate to the situation)

- a) Student discourse – Student x said/asked “ . . . ” What did that statement/question suggest to you about next steps in your instruction?
  - How did this comment/question inform you about the student’s current level of proficiency with SMPs (specific SMP if applicable)?

- Looking back on that situation, would you have proceeded differently if you had it to do again? Justify.

b) Teacher discourse – You said/asked “. . .” What were you hoping to elicit from students?

- Which SMP were you trying to guide students toward?
- Did you achieve your intended goal?
- Would you say/ask it differently if you did it again? Justify.
- Would you say/ask it differently to a different group of students? Justify.

c) Group discourse – Explain your role during this group discussion?

- What were you hoping students were attending to during that exchange?
- What evidence do you have that learning occurred?
- Which SMPs were evident in the conversation? Explain.
- How did the class norms for discourse support or hinder this interaction?

Thank you for making time to talk with me today. I look forward to observing your class and talking with you again in the near future. (Review upcoming schedule of observations if available.)

## Appendix D: Statement of Original Work

I attest that:

1. I have read, understood, and complied with all aspects of the Concordia University-Portland Academic Integrity Policy during the development and writing of this dissertation.
2. Where information and/or materials from outside sources has been used in the production of this dissertation, all information and/or materials from outside sources has been properly referenced and all permissions required for use of the information and/or materials have been obtained, in accordance with research standards outlined in the *Publication Manual of The American Psychological Association*.

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Digital Signature

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Name (Typed)

March 15, 2017

Date