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Math Vocabulary & Number Sense: Instructional Strategies for Upper Elementary English Learning Students in the Math Classroom

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MATH VOCABULARY & NUMBER SENSE: INSTRUCTIONAL STRATEGIES FOR UPPER ELEMENTARY ENGLISH LEARNING STUDENTS IN THE MATH CLASSROOM

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"The brain is like a muscle. When it is in use we feel very good. Understanding is joyous."

\- *Carl Sagan*

First and foremost, thank you to my parents, Jane and Tom, for supporting me through this journey. To my wife, Hua, for always being there when I need you most—thank you.

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ABSTRACT

Recent studies show that English learning students are a growing population within the United States school system. Language barriers have been shown to negatively influence reading scores as well as math scores, and math scores are a predictor of future academic success in many subject areas. This qualitative action research study demonstrates the difficulties many upper-elementary English learning students have understanding number sense (numeracy) in a math classroom setting in Minnesota, United States. The purpose of this study was to explore whether intentional teaching of math vocabulary could alleviate the difficulties upper-elementary English learning students have in understanding, applying, and having confidence in using number sense during math instruction in the general education classroom. In this way, the study aims to assist in overcoming the achievement and opportunity gaps impacting the English learning student population. Interview, observational, and document data were collected from upper-elementary English learning students in my math classroom during the 2022-2023 school year using purposive sampling methods. Qualitative data on vocabulary instructional methods were also collected and used to inform math vocabulary instruction throughout the school year during which the study took place. The results of this study showed that English learning students' understanding, ability, and confidence in math vocabulary improved their math performance in the classroom and decreased their misconceptions surrounding number sense. Although confidence was only improved with single-step word problems, or with a single number sense ability within a multi-step word problem, the overall findings indicate that learning math vocabulary through the use of structured dialogues contributed to greater understanding of number sense components within math word problems and increased students' ability to visualize and explain how to complete the steps needed to do in a math word problem.

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CHAPTER ONE: INTRODUCTION

Struggles for English learning students

My research explores how academic vocabulary could increase English learning (EL) students' understanding and use of number sense strategies while solving math word problems in the general classroom setting. This study examines whether using academic vocabulary in structured and unstructured settings could help EL students with learning new vocabulary, reviewing previous vocabulary, number sense components, multi-step word problem explanations, and ultimately show academic growth during the course of a school year. My interest in this area of study arose from my experiences working with EL students over the past decade, and in particular, my experience during the 2020 school year.

The COVID-19 pandemic resulted in a nation-wide drop in overall test scores for all students, including the EL student population (National Center for Education Statistics, 2022). One reason for this may be the way in which students attended school during 2020 (March - June), which for students at my school specifically, was short 10 to 15 minute phone conversations per student to cover an entire day of education. Math, language arts, science, and social studies were all done in in a total of 10 to 15 minutes. As our school staff worked with families and government agencies to obtain and provide internet access, classes were gradually moved from the telephone to video calls on the Internet. Imagine an upper-elementary EL student trying their best to learn with what little resources they had, all while navigating the new world we all were experiencing due to the COVID-19 pandemic and visualize how this new world has shaped their education. However, about 100,000 schools across the United States were closed for at least eight weeks, with 91% of schools offering a version of distance learning based on the 600 school districts sampled during 2020 (Zviedrite, 2021).

EL students struggled to keep pace with native English speakers' test scores during the COVID-19 pandemic. For example, according to the National Center for Education Statistics (NCES) (2022), fourth grade EL students scored an average of 220 on the National Assessment for Educational Progress (NAEP) Measure of Academic Progress (MAP) test for mathematics in 2019, compared to an average of 243 for non-EL students. The MAP test is an exam that measures growth instead of proficiency. A growth test adapts to the test-taker by giving more or less difficult questions based on previous responses. A proficiency test gives all test-takers the same questions and is not adaptive. This distinction is important as this study, like the MAP test, is measuring student growth throughout the course of one school year. The 23-point achievement gap correlated with 16% of fourth grade EL students testing as proficient versus 44% of non-EL fourth grade students testing as proficient. The achievement gap widened as students advanced through upper-elementary and middle school grades. The NCES (2022) reported eighth grade EL student scores on the NAEP mathematics test averaged 243 compared to an average of 285 for non-EL eighth grade students. This 42-point achievement gap correlated with 5% of eighth grade EL students testing as proficient versus 36% of non-EL eighth grade students testing as proficient.

My interest in teaching math, specifically, has increased during my teaching career as I have observed EL students struggle frequently with math concepts. During the COVID-19 pandemic, distance learning created yet another hurdle (opportunity gap) for my students, as technology and internet access was a major issue at the beginning of the school year, which caused students to fall even further behind. As I saw my students struggling to keep up with their work, I asked myself many times what I could do to help them understand the content being taught, especially new concepts in math. I tried many different strategies until I started to focus on increasing my students' understanding of the new content being taught through intentional academic-language instruction as described below.

Problem of Practice

This dissertation explores how educators could support EL students to perform at a higher level in math (in the classroom and on exams) by overcoming some of the achievement and opportunity gaps they experience using intentional academic language instruction. Most of the questions on state and national standardized math tests consist of word problems (Powell et al., 2017) and educators must plan math instruction accordingly, as there could be a gap in math vocabulary proficiency that will result in EL student misconceptions about number sense. Furthermore, the importance of math vocabulary is highlighted by the claim that the ability to explain to, argue with, and convince other mathematicians is at the heart of math itself (Boaler, 2015). This dissertation will explore the effectiveness of strengthening EL students' math vocabulary by increasing their understanding of and ability to use number sense strategies and, therefore, improving math instruction for EL students.

The topic about EL students and their math capabilities was of interest to me as my school, and most of the world, were distance learning over the phone and on computers during the beginning of the 2020 school year. One of my students, who had been flagged as a potential candidate for special education (SPED), was working with a Karen (pronounced Kah- Ren, an ethnic group of people from Southeast Asia) translator/paraprofessional and me to see if he could improve his math scores. He could understand math concepts; however, his ability to understand what he was being asked to do was lost in the translation. I hypothesized that if he understood the mathematical terms better, his math scores would increase. After working with him for an extra half-hour a day on specific math vocabulary for most of the school year, his scores on the math MAP, the national test that measures student growth, increased by ~ 40 points over the course of the school year. This result was extremely promising, as most of the students I teach achieve a growth increase of ~ 10 points during the school year in upper-elementary.

This drastic increase in his math scores made me think about all the other EL students who were struggling with math as well and how his results might be duplicated. Could more time and attention spent by upper elementary classroom teachers on math vocabulary increase EL students' understanding of number sense? How effective is teaching math vocabulary every day to an entire class to increase student confidence in their understanding of number sense? From these questions arose an assumption that math vocabulary was the key to greater number sense understanding. This assumption was addressed and tested in Chapter 2 of this study.

All students have the right to learn what they need to know to be successful in life. For an equitable education, EL students need to be taught academic vocabulary in order for them to grasp the concepts being taught in class. My student who gained ~ 40 points would not have made these gains if he were not intentionally taught academic vocabulary for math; at least that is my hypothesis. To explore whether my hypotheses about this student were correct or not, I conducted a study with upper-elementary math students in my classroom during the 2022-2023 school year and recorded the results of students learning vocabulary and their ability to use number sense strategies along with students' confidence in their answers, strategies used, and math vocabulary.

Study Purpose

The purpose of this study is to explore whether intentional teaching math vocabulary using spiral academic techniques has any impact on alleviating the difficulties upper-elementary EL students have in understanding, applying, and having confidence in using number sense during general education math instruction. Spiral teaching, or spiral review, is where students are retaught or given review problems for content taught in previous grades or previously during the school year. For this study, number sense is defined as, "A propensity for and an ability to use numbers and quantitative methods as a means of communicating, processing, and interpreting information. It results in an expectation that numbers are useful, and that mathematics has a certain regularity (makes sense)" (McIntosh et al., 1992). In this study, I identify best practices for teaching math and math vocabulary to EL students who general education teachers could implement in the classroom during whole-group instruction, instead of having students pulled out of the classroom during core instruction time for language instruction, Title 1, or any other form of small group instruction.

Research Questions

This study was guided by the following research questions:

Central question: What role, if any, does math vocabulary instruction have in increasing upper elementary students' confidence in their understanding and application of number sense strategies?

Sub questions:

1) What role does grade-level math vocabulary instruction play in enhancing the confidence level and use of number sense strategies of upper elementary EL students?
2) What role does spiral math vocabulary instruction play in enhancing the confidence level and use of number sense strategies of upper elementary EL students?
3) How do upper-elementary EL students perceive that math vocabulary instruction contributes to their understanding of early number sense strategies?

4) How do upper-elementary EL students perceive that math vocabulary instruction contributes to their understanding of mature number sense strategies?

Significance of the Study

English learning (EL) students are a growing population in the United States public school system (Kung et al., 2021). Recent studies showed that the English language barrier faced by EL students not only negatively influenced reading scores, but math scores as well (Aguirre-Muñoz & Boscardin, 2008; Cook et al., 2011; Doabler et al., 2019). Multiple studies (Henry et al., 2014; Jordan et al., 2002; Zakaria & Aziz, 2011) have identified the link between English language proficiency and math achievement when the language of instruction is English. As math scores can be an early predictor of future academic success for all students in areas other than math (Purpura et al., 2017; Peng & Lin, 2019), increasing EL students math abilities is an urgent matter of study.

The contribution to the knowledge and practice of teaching math to EL students is emphasizing the importance of intentionally teaching domain-specific vocabulary to students during classroom instruction in a general education setting. Scholars already knew that linking language and content instruction was an effective teaching practice (Schleppegrell, 2007). However, more research is needed to determine the impact of scaffolding domain-specific vocabulary in all content areas (Nagy & Townsend, 2012). This study contributes to the scholarly conversation surrounding linking language and content instruction by researching the scaffolding of domain-specific, spiral, and general academic vocabulary instruction for EL students in the general education math classroom.

Research Site and Participants

This study took place at an urban school in the state of Minnesota in the United States in my classroom during the 2022-2023 school year using action research. The school was selected for this study because I had access to the students in my classroom who were participants in the study with their parents' approval, support of the school administration for the study, and the student population was mostly EL students. I taught math and science to two sections of upper-elementary students with a total of 29 students, 12 of whom participated in the study. As each school district and school designs their own learning periods, which causes variation in the amount of instructional time, it will be noted that each class I taught received at least 70 minutes of math instruction per day on average.

The school had a total enrollment of 445 students in grades preK-8 with 100% receiving free or reduced lunch and many of the students (84%) identified as Asian during the year of this study. The ethnicity of students identifying as Karen, Hmong, or other was not collected school-wide, however, for the 12 students who participated in the study, nine students identified as Karen and three students identified as Hmong. The percentage of students classified as English learners during the 2022-2023 school year was 39%, however this percentage of the student population does not include students who tested out of the EL program prior to the school year via the WIDA ACCESS test, which is used by 41 U.S. states and territories (WIDA, 2020). For the 12 students in my study, six students were classified as being EL while the six students not classified as EL had exited the program during their tenure at the school prior to the 2022-2023 school year. Students who have officially exited the EL program were included in the study as EL learners because WIDA ACCESS does not test specifically for math vocabulary. Math items are embedded in some sections of the test, but not in its entirety (S. Westra, personal communication, July 13, 2023).

Definition of Key Terms

The particular context in which my study took place entails the use of specific terms. The definitions that I used in this dissertation are explained below.

English learner and English learning students: “Students who are unable to communicate fluently or learn effectively in English, who often come from non-English-speaking homes and backgrounds, and who typically require specialized or modified instruction in both the English language and in their academic courses” (Glossary of Education Reform, 2013).

Hmong: An ethnic group of people from Southeast Asia who started coming to Minnesota as refugees in 1975 (MN Historical Society, 2022).

Karen: An ethnic group of people, mainly from Myanmar/Burma. The Karen people started resettling in Minnesota after fleeing to Thailand and living in refugee camps (Karen Organization of MN, 2022).

Number sense: "A propensity for and an ability to use numbers and quantitative methods as a means of communicating, processing, and interpreting information. It results in an expectation that numbers are useful, and that mathematics has a certain regularity (makes sense)" (McIntosh et al., 1992, p. 4).

Position of Researcher

While operating with an interpretivist lens, I stayed grounded in my mission statement when planning and conducting my research, which is: *Continually striving for evolution of self, organization, and community through education, clarity, and human connectedness.* Action research seamlessly integrated with my regular classroom practices. The data collected was from student classwork, my own observations, and student interviews. Action research is a cyclical process of systematic inquiry conducted by people within a community with a goal of identifying practices that generate improvement that is meaningful and important (Hinchey, 2008). Through action research, my role as a researcher was to inform and influence my role as a teacher to improve the way I teach and the way my organization applies the knowledge learned about how EL students perceive and understand the world of mathematics.

While conducting my research, I was considered an *insider* as opposed to an *outsider*. Being an *insider* in the realm of action research means that I was researching my own practice, or practice setting (Herr & Anderson, 2014). I was not studying myself, but rather I was studying the outcome of actions taken in my own setting. Those actions included the intentional teaching of academic language during math class and the collection of student data to determine if student number sense abilities and confidence were impacted. This research differs from research as an *outsider*, as *outsiders* study *insiders* in a setting that is unfamiliar to the *outsider* (Herr & Anderson, 2014). My position as an *insider* researching an action in my own classroom did not have a significant effect on my students.

For my positionality as a teacher, my experience teaching EL students for most of my teaching career has shaped how I view education. I view education as continuous improvement and in order to nurture student growth, teachers and organizations must grow in their knowledge and understanding of their students' reality. There are multiple realities or truths and to fully understand the world of education, I must use the knowledge gained from dialogue with participants to find practical solutions. The practical solution for the purpose of this study being a more effective way to teach upper elementary EL students number sense.

Overview of Previous Research

There has been extensive research on EL teaching strategies, such as who EL students are (Cummins, 1986) and how EL students best learn (Cummins & Mann, 2007; Haas & Brown, 2019; Lestari & Jailani, 2018; Muhid et al., 2020). However, there are not many studies on EL instruction in the area of math. There is also extensive research on teaching vocabulary and how students best learn it (Anstrom et al., 2010; Nagy & Townsend, 2012), but only a few studies focus on math vocabulary. Research on number sense is a new area of study; there are multiple meanings, terms, and constructs that scholars are unable to fully agree on (Whitacre et al., 2020). My study centers on math instruction, math vocabulary instruction, and number sense constructs to add to the research in these areas for EL students.

Increasing EL students' number sense could be possible with the use of many differing strategies such as scaffolding (teaching a lesson or problem by steps or strategies), allowing extra time, providing many opportunities to learn a new concept, modeling new content using your work as well as previous student work, and teaching about metacognition to increase EL students' self-efficacy. Many authors have noted that metacognition and self-efficacy were essential for EL students (Bandura, 1997; Haas & Brown, 2019; Lestari & Jailani, 2018; Muhid et al., 2020). Due to the lack of research in math vocabulary, this study will focus on the instructional practice of teaching new vocabulary along with new math concepts to help EL students grow their math vocabulary skills to enhance student understanding of and confidence in number sense strategies.

Scholars have identified intensive vocabulary instruction as an additional best practice in EL students' education (Haas & Brown, 2019; Helman, 2008; Lestari & Jailani, 2018). Vocabulary instruction may take different forms based on what type of vocabulary is being taught. This distinction of vocabulary types is important for teachers to understand in order to best teach students. Research involving vocabulary instruction for math is fairly new, but a newer field of study is research on number sense and what it is. Despite all the existing research on EL teaching strategies, math vocabulary, and number sense, there is little specific research on upper-elementary EL students' math vocabulary and number sense. My study proposes to identify an area of research applicable to EL teaching, math vocabulary, and understanding number sense to better serve upper-elementary EL students' understanding of core math concepts.

Conceptual Framework

The framework for this study is centered on the key components of number sense, academic vocabulary, and domain-specific language. Number sense is defined as, "A propensity for and an ability to use numbers and quantitative methods as a means of communicating, processing, and interpreting information. It results in an expectation that numbers are useful, and that mathematics has a certain regularity (makes sense)" (McIntosh et al., 1992, p. 4). I used this definition because it encapsulates the notion that math has meaning and can be used to communicate. Communication using language can be separated into two main categories, basic interpersonal communication skills (BICS) and cognitive academic language proficiency (CALP) (Cummins, 1979). Cummins theorized a difference between social and academic language, however academic language can be further segmented into three types of vocabulary: general, generative, and domain-specific (Helman et al., 2017). General academic vocabulary includes words and phrases that may be used across content areas but are not generally used in basic interpersonal communication. Generative vocabulary is the ability to decipher words based on knowledge of the English language system, i.e., root words, prefixes and suffixes. Domain-specific vocabulary is defined as nouns, specific to one area of study, that have an exact meaning and are not used in any other way outside of the content area (Helman et al., 2017). These frameworks on different forms of communication inform the conceptual framework for my study, which posits that explicit math vocabulary instruction can be paired with specific number sense strategies to increase the student's ability to use, comprehend, and communicate number sense strategies. This framework aligns with Schleppegrell's (2007) stance that math and language must be taught simultaneously. For example, when learning about fractions, students need to have the number sense abilities of operating on fractions/decimals and an understanding of the relationship between operations, such as multiplication and division being the inverse of each other. Vocabulary for these number sense abilities would include, but is not limited to inverse, product, quotient, denominator, numerator, etc.

Conclusion

In this chapter I have presented the problem of practice fueling this study, that is, increasing the student's number sense confidence and abilities through the intentional teaching of academic vocabulary. For this study, I used an action research approach in my own math classroom during the 2022-2023 school year. The student population participating in the study were upper elementary EL students. The conceptual framework for my study synthesizes research on EL teaching strategies, what academic language is, what effective vocabulary instruction looks like, and number sense constructs to explore teaching practices that improve EL students' understanding of number sense. This study is significant because of the opportunity and achievement gaps experienced by the EL student population. The sequence of my dissertation will start with a review of relevant literature, followed by methodologies, data analysis, and recommendations. In the next chapter, I will present relevant literature on the topics of number sense, achievement & opportunity gaps, EL education, and academic vocabulary.

CHAPTER TWO: REVIEW OF LITERATURE

Introduction

The following review of literature begins with a perusal of the rapid growth of the English learning (EL) student population in the United States public school system, illuminating the need for transforming instructional practices. The terms English learning student(s) and English learner(s) will be used interchangeably throughout this literature review. Other terms for EL students who were not chosen for this study include Multilingual learners (ML), English language learners (ELL), and English as a second language (ESL) students. The term “ML” describes students actively developing proficiency in more than one language, which may apply to some but not all of the students participating in this study. The term ESL was used to describe a group of people in the past, but now it is more affiliated with instructional courses rather than people (National Council of Teachers of English, 2017). The term ELL, while having no technical difference to EL, is being phased out in many states (National Council of Teachers of English, 2017). The Minnesota Department of Education exclusively uses the term EL to refer to English learning students on their educational websites (Minnesota Department of Education, n.d.).

My dissertation sought to understand how EL students' confidence level and ability to correctly identify and use number sense strategies increases through intentional math vocabulary instruction, as domain-specific academic vocabulary instruction has a need for further research (Nagy & Townsend, 2012). In preparation for this study, I asked: How have scholars understood the relationship between language acquisition, number sense ability, and confidence with respect to EL students? To answer this question, I examined three bodies of literature.

The first body, *number sense*, describes the many different constructs and subconstructs of number sense; also known as numeracy. Scholars referenced in this body of literature describe the ability to use, interpret, and communicate numbers. However, not all scholars can agree how to group those abilities. The second body, *opportunity gaps, achievement gaps, & self-efficacy*, describes strategies of best practices concerning the education of EL students. Scholars detail the documented achievement gap impacting EL students as being an outcome of the opportunity gap experienced by EL students and describe teaching practices to best support EL students in the classroom. The third body, *vocabulary and language acquisition*, describes how students acquire language, the characteristics and types of vocabulary that comprise the vocabulary of academic language, and the importance of vocabulary learning in all areas, or subjects, of academics for EL students.

Therefore, the goal of my dissertation is to add to the scholarly conversation concerning math language acquisition curriculum for EL students implemented in the general education classroom. This literature review will provide the conceptual framework of my research to further the understanding of EL student math education. To provide context for the literature review, I commence with an overview of the history of EL education in the United States and the state of Minnesota followed by a brief synopsis of inadequacies in EL education.

EL Education in the United States: The History and Context

English learning (EL) students are a growing population in the United States public school system (Kung et al., 2021; Nieto, 2009). During the decade comprising the 1990s, the EL student population grew by 52 percent (Fix & Passel, 2003). The EL student population in the public school system varies from state to state, however, during the autumn of 2019, the EL student population was 10.4 percent of all students in the United States public school system according to the National Center for Education Statistics (NCES) (2022), and the EL population is expected to rise to 25 percent nationally by the year 2050 (Nieto, 2009). In Minnesota, the EL population in the public school system is 8.6 percent (NCES, 2022).

The development of EL education in the United States began in 1968 with the Bilingual Education Act, which did not require schools to provide bilingual programs, however it encouraged experimentation on new pedagogical approaches aimed at non-English speaking populations with supplemental federal funding (Nieto, 2009). In 1974, the Bilingual Education Act was amended to require progress reports and the Supreme Court ruled it was school districts' responsibility to provide programs and accommodations for non-English speaking students (Nieto, 2009). During the Reagan administration era, bilingual education was scrutinized and set-back until the Bilingual Education Act was reauthorized under the Improving America's Schools Act in 1994 (Nieto, 2009).

Each state has its own laws concerning EL education. In Minnesota, that law is the Learning English for Academic Proficiency and Success (LEAPS) Act, which was passed in 2014. The LEAPS Act covers many areas and grade levels concerning EL education and requires (with no real penalty for not obtaining this goal) that all students, regardless of EL status, read at grade level in English by the end of third grade (Office of the Revisor of Statutes, 2014). The LEAPS Act also states that schools and teachers are responsible for planning, implementing, differentiating instruction, and evaluating students to reach this goal, with one of the required focus areas being vocabulary instruction. Vocabulary is part of the five reading components of phonemic awareness that also includes phonics, phonemic awareness, fluency, and comprehension.

In 2015, the Obama administration signed the Every Student Succeeds Act (ESSA) into law, and the law was revised in 2017 by the Trump administration (Department of Education, 2022). The ESSA requires each individual state to inform the U.S. Department of Education their plans to support EL students' learning and plans to support teachers with training to better assist EL students. Since EL teacher training and instructional requirements are the responsibility of each individual state (US Department of Education, 2022), and in Minnesota, teacher training and instructional requirements are the responsibility of each individual district, school, and teacher (Office of the Revisor of Statutes, 2014); there is no clear direction of EL education on the federal or state level. The lack of clear direction for EL education may be a factor leading to inadequate teacher training and student instruction (Li, 2018; Shim & Shur, 2018). Examples of these inadequacies include classroom instruction being disrupted for EL students being pulled out of class for small-group instruction (Theoharis & O'Toole, 2011), causing teachers to cover more material in a shorter amount of time (Nelson et al., 2020), leading to students not being given enough time to learn new concepts and causing EL students to feel misunderstood and disrespected (Theoharis & O'Toole, 2011). These inadequacies are compounded by EL students not receiving adequate instruction in the separate EL classrooms (Shim & Shur, 2018; Theoharis & O'Toole, 2011), as core materials may differ from intervention materials, causing further confusion for EL students (Nelson et al., 2020). This leads to another source of confusion surrounding education; understanding what number sense is and is not.

Number Sense

In this first body of literature, I discuss the concept of number sense, or numeracy as some scholars refer to it, which is defined as, "A propensity for and an ability to use numbers and quantitative methods as a means of communicating, processing, and interpreting information. It results in an expectation that numbers are useful, and that mathematics has a certain regularity (makes sense)" (McIntosh et al., 1992, p. 4). Number sense is not about memorization of facts and procedures; rather, it is about the mindset and the process of thinking about numbers and quantities (Boaler, 2015). Number sense is important to my study because it is what EL students are struggling with in my school and nationwide. Educators must know what specific components of number sense students are struggling with to introduce the correlating domain-specific vocabulary. Educators must also know what components of number sense are linked together and in what order for spiral domain-specific vocabulary instruction in math.

To this day, scholars do not agree on the breakdown of what number sense is and where one construct ends and another begins, which causes confusion and a perceived reversal of research (Whitacre et al., 2020). The more the constructs that make up number sense are studied, the less researchers know about how the constructs are related (Whitacre et al., 2020). This body of literature will give examples, define differing number sense constructs, and underline the overlapping of various constructs to show why there are current discrepancies in the number sense field of study. The three constructs of number sense are early number sense, foundational number sense, and mature number sense.

The understanding of these constructs, how they overlap, and the distinctions between them is needed to answer the question: How have scholars understood the relationship between language acquisition, number sense ability, and confidence concerning EL students? The goal of this literature review is to establish the relationship between the English language and mathematics. An understanding of what scholars identify as math constructs, from a conceptual standpoint, must be obtained to understand how language correlates with the constructs as well as understanding of how and why my study will be organized in my classroom.

Whitacre et al. (2020) coded 124 articles that research and define the parameters of number sense prior to 2016. The Whitacre study identified three main constructs that make up the current research on number sense: approximate number sense (ANS), early number sense (ENS), and mature number sense (MNS) (2020). Even though Whitacre et al. (2020) identified three number constructs, number sense constructs are not limited to three terms as there are many terms used by various scholars. One of those terms, foundational number sense (FONS) identified by Andrews & Sayers (2015), will be discussed as well to display the overlapping of the ENS and MNS constructs by scholars.

This compounding of components can cause inadequacies in educator instructional strategies concerning EL students (Yang & Lin, 2015) on the breadth and depth of potential unknown vocabulary (Helman et al., 2017) as well as a lack of using EL teaching strategies (Li, 2018). Educators' ability to understand what number sense is and use of the current knowledge of competing theoretical constructs (ENS, FONS, MNS) as a roadmap to identify student misconceptions can lead to better-informed instruction (Anghileri, 2000) using current language acquisition strategies (Riccomini et al., 2015).

The first number sense construct identified by Whitacre et al. (2020), ANS, is applicable to infants as well as to some animals. Aside from one ANS component being introduced in kindergarten math standards, ANS does not have any significant overlap with the other constructs and is not relevant to upper-elementary students. Therefore, ANS will not be further discussed in this literature review. The number sense constructs that pertain to upper-elementary-aged students, as well as adults, are those of ENS and MNS identified by Whitacre et al. (2020), and FONS identified by Andrews & Sayers (2015). ENS involves number recognition, number patterns, counting, estimation, number operations, and number comparison; while MNS involves mental computation, selection of strategy, flexibility, and reasonableness of the solution (Whitacre et al., 2020). Research concerning MNS will be the focus of this literature review as the populations studied for MNS are mainly elementary-aged children and older (Whitacre et al., 2020). However, with the considerable overlap between differing number sense constructs in the research community, I will also discuss ENS and FONS research.

For this literature review, I used the Whitaker et al (2020) typology to compare three studies of number sense to demonstrate the overlap of constructs within the research community. From McIntosh et al. (1992) study, number sense is divided into the three main components of *numbers*, *operations*, and *computational settings*, which are then divided into subcomponents that are subsequently divided into base components (30 in total) as depicted in Table 1 (McIntosh et al., 1992, p. 4). Whitacre et al. (2020) coded, or classified, McIntosh et al. (1992) as falling in the MNS construct.

For the second study, Andrews & Sayers (2015) developed a framework for foundational number sense (FONS) by identifying eight components that were used to analyze learning opportunities for students in varying cultural contexts and was coded as ENS by Whitacre et al. (2020). I compared each component identified by Andrews & Sayers (2015) to the 30 base components identified by McIntosh et al. (1992) for this literature review and they are listed in Table 2.

Table 1

Mature Number Sense (McIntosh, et al., 1992)

Number Sense: A propensity for and an ability to use numbers and quantitative methods as a means of communicating, processing and interpreting information. It results in an expectation that numbers are useful and that mathematics has a certain regularity (makes sense).	1 Knowledge of and facility with NUMBERS.	1.1 Sense of orderliness of numbers	1.1.1 Place value 1.1.2 Relationship between number types 1.1.3 Ordering numbers within and among number types
		1.2 Multiple representations for numbers	1.2.1 Graphical/symbolic 1.2.2 Equivalent numerical forms (including decomposition/recomposition) 1.2.3 Comparison to benchmarks
		1.3 Sense of relative and absolute magnitude of numbers	1.3.1 Comparing to physical referent 1.3.2 Comparing to mathematical referent
		1.4 System of benchmarks	1.4.1 Mathematical 1.4.2 Personal
	2. Knowledge of facility with OPERATIONS.	2.1 Understanding the effect of operations	2.1.1 Operating on whole numbers 2.1.2 Operating on fractions/decimals
		2.2 Understanding mathematical properties	2.2.1 Commutativity 2.2.2 Associativity 2.2.3 Distributivity 2.2.4 Identities 2.2.5 Inverses
		2.3 Understanding the relationship between operations	2.3.1 Addition/Multiplication 2.3.2 Subtraction/Division 2.3.3 Addition/Subtraction 2.3.4 Multiplication/Division
	3. Applying knowledge of and facility with numbers and operations to COMPUTATIONAL SETTINGS.	3.1 Understanding the relationship between problem context and the necessary computation	3.1.1 Recognize data as exact or approximate 3.1.2 Awareness that solutions may be exact or approximate
		3.2 Awareness that multiple strategies exist	3.2.1 Ability to create and/or invent strategies. 3.2.2 Ability to apply different strategies 3.2.3 Ability to select an efficient strategy
		3.3 Inclination to utilize an efficient representation and/or method	3.3.1 Facility with various methods (mental, calculator, paper/pencil) 3.3.2 Facility choosing efficient number(s)
		3.4 Inclination to review data and result for sensibility	3.4.1 Recognize reasonableness of data 3.4.2 Recognize reasonableness of calculation

The six number sense components developed by Andrews & Sayers (2015) listed in Table 2 are the ENS components that overlap with the MNS sub and base components listed by Whitacre et al. (2020). Classification of these components is uncertain as an argument could be made for each component to be classified as either ENS or MNS. The eight number sense components determined by Andrews & Sayers (2015) could also be placed as sub-base components of the McIntosh et al. (1992) base components. Neither *number recognition or awareness of relationship between number and quantity* components are listed by McIntosh et al. (1992), and therefore, can represent examples of ENS components for this literature review.

Table 2

A Comparison of ENS & MNS Constructs

Coded ENS: Andrews & Sayers (2015)	**Coded MNS: McIntosh et al. (1992)**
Systemic counting =	Ordering numbers within and among number types (1.1.3)
Quantity discrimination =	Comparing to physical referent (1.3.1)
Understanding different representations of numbers =	Multiple representations for numbers (1.2)
Estimation =	Recognize data as exact or approximate (3.1.1)
Simple arithmetic =	Operating on whole numbers (2.1.1)
Awareness of number patterns =	Ordering numbers within and among number types (1.1.3) Recognize reasonableness of data (3.4.1)

The third study I compared for this body of literature was written by Yang & Lin (2015), who identified five number sense components, which are the basics of what number sense is, as described briefly by McIntosh et al. (1992). The first three components identified by Yang & Lin (2015) are *understanding the meanings of numbers and operations, recognizing the number size*, and *using multiple representations of numbers and operations* overlap with the first two subcomponents identified by McIntosh et al. (1992). The final two components, *recognizing the relative effect of operations on numbers* and *judging the reasonableness of computational results*, overlap with McIntosh et al. (1992) second and third subcomponents. Whitacre et al. (2020) coded Yang & Lin (2015) as falling in the MNS construct, matching their coding with McIntosh et al. (1992). The study done by Yang & Lin (2015) has the vaguest number sense components of the three studies, however, how the research was conducted is the reason this study is being critiqued.

Yang & Lin (2015) developed a four-tier diagnostic test for this quantitative study based on an earlier two-tier diagnostic test to measure student number sense, confidence levels, and misconceptions of 195 fifth-grade students in Taiwan. One notable result of the study showed that students had a significantly higher confidence rating for their answer than for the reason why they chose the answer. This shows that students may understand how to calculate a problem, but do not understand the mathematical rules for why those calculations produce a correct answer (Yang & Lin, 2015). Using this study's instrument with more precise number sense components identified by McIntosh et al. (1992) and Andrews & Sayers (2015) may reveal a greater understanding of student misconceptions concerning number sense and will serve as an instrument for my study. I used number sense components in the FONS, ENS, MNS & ANS constructs to measure student confidence and ability in number sense strategies and the domain-specific vocabulary that correlates with those components.

Correlating State Standards & Number Sense

This first body of literature has explained what number sense is and identified the overlapping of different number sense constructs. As students progress through the K-12 public education system, number sense abilities increase in difficulty, shifting constructs from ENS to MNS, with the addition of compounding number sense components and subcomponents in math instruction (Minnesota Department of Education, 2022). For example, as depicted in Table 3, when comparing the second-grade math standard of solving real-world problems using addition and subtraction on whole numbers (2.1.2.5) to the fifth grade math standard of solving real-world problems using addition and subtraction on decimals, fractions, and mixed numbers (5.1.3.4), this compounding of number sense components within the MNS construct can be identified.

The second-grade standard contains two number sense components (McIntosh et al., 1992): *operating on whole numbers* (2.1.1) and *comparing to physical referent* (1.3.1). The fifth-grade standard builds on the two number sense components in the second-grade standard by incorporating the number sense components of: *operating on fractions/decimals* (2.1.2), *multiple representations for numbers* (1.2), and *relationship between number types* (1.1.2). Understanding number sense and its constructs should assist educators in identifying student misconceptions on specific areas of number sense.

To reiterate: This compounding of components can cause inadequacies in educator instructional strategies for EL students (Yang & Lin, 2015) on the breadth and depth of potentially unknown vocabulary (Helman et al., 2017) as well as a lack of using EL teaching strategies (Li, 2018). Educators' ability to use number sense and current knowledge of competing theoretical constructs as a roadmap to identify student misconceptions can lead to better-informed instruction (Anghileri, 2000) by using current language acquisition strategies (Riccomini et al., 2015). The next body of literature will review teaching strategies for EL students that may assist in the instruction of number sense to increase student number sense abilities and confidence.

Table 3

Example of Compounding Number Sense Components in Math Standards

2nd grade Math Standard Solving real-world problems using addition and subtraction on whole numbers.	**5th grade Math Standard** Solving real-world problems using addition and subtraction on decimals, fractions, and mixed numbers.
• *operating on whole numbers* (2.1.1) • *comparing to physical referent* (1.3.1)	• *operating on whole numbers* (2.1.1) • *comparing to physical referent* (1.3.1) • *operating on fractions/decimals* (2.1.2) • *multiple representations for numbers* (1.2) • *relationship between number types* (1.1.2)

Opportunity Gaps, Achievement Gaps, & Self Efficacy

This body of literature focuses on current research to review EL teaching strategies and relate those strategies to math instruction. First, misconceptions of the EL student population in the United States must be addressed. Misconceptions regarding the EL student population as not being as capable as non-EL students have been well documented in recent years (Rizzuto, 2017; Shim & Shur, 2018; Theoharis & O’Toole, 2011), especially when considering national standardized tests. According to the National Center for Education Statistics (NCES) (2022), fourth grade EL students scored an average of 220 on the National Assessment for Educational Progress (NAEP) for math compared to an average of 243 for non-EL students. This 23-point achievement gap correlates with 16% of fourth grade EL students testing as proficient versus 44% of non-EL fourth grade students testing as proficient.

The achievement gap widens as students advance through upper-elementary and middle school grades. The NCES (2022) reported eighth grade EL student scores on the NAEP math test averaging 243 compared to an average of 285 for non-EL eighth grade students. This 42-point achievement gap correlates with 5% of eight grade EL students testing as proficient versus 36% of non-EL eighth grade students testing as proficient.

The discussion surrounding the achievement gap of EL students has shifted to an opportunity gap discussion in recent years. There is an argument concerning the significance of home life and socioeconomic status as being contributing factors to this opportunity gap (Henry et al., 2014; Kung et al., 2021). It has also been widely documented that standardized tests in America are culturally and socially biased (Salend et al., 2002). While these factors indeed have an impact on many EL students, instructional practices in the classroom may have a profound impact on student opportunities and achievement as well (Rizzuto, 2017), leading to the misconceptions of EL student capabilities. Whether or not it is purely the instructional practices leading to misconceptions of EL students, or if misconceptions of EL students lead to instructional practices, or a mixture of both; it is instructional practices in the classroom that will be the focus of the remainder of this chapter.

An example of an opportunity in the classroom that EL students are not always perceived as being capable of, depending on their developmental stage, is to showcase their knowledge and/or learning by manipulating language in order to represent their thinking with academic concepts (Haas & Brown, 2019). While many EL students can manipulate language to represent their thinking within a social context with conversational language (Cummins & Man, 2007), the same cannot be said for academic language. If EL students are not explicitly taught academic language, then this opportunity to represent their thinking may be lost to them, and EL students may be perceived as being less educated and being incapable of performing an assigned academic task by their teacher or school (Barwell et al., 2015; Shleppegrell, 2012). This lost opportunity disempowers and puts the blame on EL students for failing to take advantage of an opportunity they did not receive, and creates misconceptions of EL students of being less capable as well as a cascade of other lost opportunities, such as learning the assignment and showcasing their knowledge on homework and exams (Cummins, 1986).

Examples of misconceptions of EL students include the benchmark of all students reading on level by third grade created by Minnesota. This particular misconception assumes EL students will be able to master the English language several years faster than studies show EL students are capable of doing (Cummins & Man, 2007; Haas & Brown, 2019). Since it takes several years for students to comprehend the English language, EL students may not be able to convey their thinking by manipulating language (Haas & Brown, 2019), which will cause them to seem less capable. Since EL teacher training and instructional requirements are the responsibility of each individual state (Department of Education, 2022), and in Minnesota, teacher training and instructional requirements are the responsibility of each district, school, and teacher; there is no clear direction of EL education, leading to the identified inadequate teacher training on EL education (Li, 2018).

There are several instructional practices that could be utilized to increase the opportunities provided for EL students. The first half of this body of literature laid out the achievement and opportunity gaps in the classroom. Since multiple studies (Henry et al., 2014; Jordan et al., 2002; Zakaria & Aziz, 2011) identify the link between English language proficiency and math achievement when the language of instruction is in English, the second half of this body of literature will identify and describe EL teaching strategies to increase the students' confidence and ability with a focus on learning through multiple repetitions of language instruction. It is imperative to know the level of English proficiency by gauging each student's characteristics, how they best gain skills in a new language, what they currently understand, and what they are currently able to do (Haas & Brown, 2019) to understand the strategies that work best for each specific student.

Metacognitive Strategies

One strategy for accelerating EL student literacy development is purposefully teaching students about how their brains work and how to understand their own way of thinking (Haas & Brown, 2019), a strategy known as metacognition. The teaching of metacognitive strategies to EL students was determined to have a significant positive effect on learning outcomes (Lestari & Jailani, 2018; Muhid et al., 2020). Since EL students have a positive response to metacognitive strategies, instruction encapsulating making inferences, summarizing, monitoring, and questioning can counteract EL students struggling to use other various learning strategies presented in the classroom (Haas & Brown, 2019).

In the math classroom, metacognition strategies may take on the form of questioning, inquiring, reflection, critical thinking, and self-explanation (Lestari & Jailani, 2018). As discussed in the first body of literature, the student's confidence levels pertaining to explaining why a chosen strategy works for solving a certain math problem is relevant to the student's understanding and being able to correctly solve a problem (Yang & Lin, 2015). This, along with number sense involving communicating, processing, and interpreting information (McIntosh et al., 1992), underscores metacognition's integrality in EL students' understanding of number sense.

The central metacognitive strategy of self-explanation requires students to use related metacognitive strategies to describe the mathematical strategies used to solve a given math problem (Lestari & Jailani, 2018), echoing the findings of Yang & Lin (2015). Metacognitive strategies should be embedded in regular classroom instruction as best practice strategies (Lestari & Jailani, 2018), furthering the argument for the practice of keeping EL students in the classroom during core instruction time (Haas & Brown, 2019). Removing EL students from the classroom during the day for EL instruction in a smaller setting causes classroom disruptions as well as diverges from the core content being taught in the classroom (Theoharis & O'Toole, 2011).

Instruction That Leads to Self-Efficacy

Metacognition can be a gateway for students to gain self-efficacy, as students who use task-related metacognitive strategies gain knowledge and skills via self-regulation (Caprara et al., 2008). Metacognition is the ability to think about one's own thinking; self-regulation is the ability to control one's own thinking; and self-efficacy is the belief in one's ability to control their own thinking, which leads to motivation (Bandura, 1994). Without the ability to control your own thinking and be confidently in control of your own thinking, the ability to think about your own thinking is inconsequential (Caprara et al., 2008). This section will explore how working towards mastery of the English language leads to higher self-efficacy and how self-efficacy for EL students is integral to their future success (Soland & Sandilos, 2021).

There are four influential processes for building self-efficacy, which include cognitive, motivational, affective, and selection processes. In addition to the influential processes, there are four main sources of influence (Bandura, 1994). The first source of influence for self-efficacy is through mastery experiences. Failing at a task and persevering through the challenge builds resilience and an ability to overcome future obstacles. However, if a task is too difficult, it may damage a person's self-efficacy. This leads to the second source of influence for self-efficacy, which is social persuasion. The structuring or scaffolding of situations, or lessons, can strengthen a person's self-efficacy through measured self-improvement (Bandura, 1994). Scaffolding in education is slowly handing over responsibility to a child or student through what amounts to increased rigor (Walqui, 2006). EL students need extra scaffolding for lessons as they are learning both content and language components. Among other scaffolding strategies, educators should model each step of the thought process to solve a math problem (Walqui, 2006).

Social persuasion can be positively reinforced through social models, which is the third source of influence for self-efficacy (Bandura, 1994). Scaffolding occurs when a model influences the actions of another (Walqui, 2006). For a model (an educator) to be sufficiently persuasive and influence the audience (students) to gain perception of their own self-efficacy, there must be perceived similarities between the audience and the model about which the audience aspires to be more like (Bandura, 1994). Educators are more successful teaching their students about content and increasing the students' self-efficacy when they can form interpersonal relationships built on trust and view each other as fellow individual human beings

(Frymier & Houser, 2000). EL students' self-efficacy and test scores also increase when students perceive their teacher to be caring and supportive (Soland & Sandilos, 2021). However, educators themselves do not always need to be the model. Using a student's work from previous classes to model a lesson can show EL students that someone just like them can succeed at the lesson (Walqui, 2006).

The fourth source of influence for self-efficacy is to explicitly teach about controlling and interpreting emotional and physical responses to stress. People with higher self-efficacy have an ability to interpret and control these body responses as an energizing boost to use on the task at hand (Bandura, 1994). Teaching about controlling emotional and physical responses can be implicit as well. Implicitly, a teacher can positively influence the mindset of a student by praising students for their efforts rather than their intelligence, expecting high standards, and providing a nurturing environment (Dweck, 2006) where students feel safe to take risks (Frymier & Houser, 2000).

Since the students' confidence levels correlate to students having the ability to explain why a chosen strategy works for solving a certain math problem (Yang & Lin, 2015), self-efficacy instruction is pertinent for EL students in the math classroom. As with self-efficacy increasing student ability in solving math problems, the same holds true for other content areas. As EL students progress in their learning of the English language, they become more confident in their abilities to complete tasks using the English language (Soland & Sandilos, 2020).

This body of literature has identified misconceptions about EL students, such as that they are not as capable, that opportunity gaps (in school) lead to achievement gaps that lead to opportunity gaps (post-education), and teaching strategies to raise EL students' confidence levels and increase students' ability in the classroom through measured success in overcoming those gaps. Many scholars have identified the importance of metacognition for EL students (Bandura, 1994; Haas & Brown, 2019; Lestari & Jailani, 2018; Muhid et al., 2020); therefore, many of the strategies presented in this section center on the importance of EL students being able to understand how their own brain works, how to control it, and how to increase their confidence level in controlling it. Self-efficacy, in its own definition, is being confident in oneself to complete a certain task. The final body of literature examines one more EL teaching strategy: the intentional instruction of subject-specific English with a focus on language acquisition and math vocabulary.

Vocabulary & Language Acquisition

This body of literature focuses on best practices identified by scholars to assist EL students in the acquisition of the English language. This process is vital for EL students to become more confident in completing tasks, including math problems, and using the English language. To begin this body of literature, a point must be made to separate conversational English language from academic language. Many EL students may sound proficient due to their knowledge of conversational English considering their ability to quickly develop initial informal oral fluency (Haas & Brown, 2019). However, even though EL students possess the ability for sustained conversation using informal English, EL students still need several years, five or more, of language instruction to gain academic language fluency (Cummins & Man, 2007; Haas & Brown, 2019). After academic language is introduced, defined, and dissected, an exploration of the literature pertaining to scholarly insights on how to best increase EL student registers concerning math vocabulary through linking language and content learning and explicit vocabulary instruction will ensue.

Academic Language

Much like the research concerning number sense, scholars are not able to agree on the boundaries of what constitutes academic language (Anstrom et al., 2010; Nagy & Townsend, 2012). Some scholars define academic language as “the language used in school to help students acquire and use knowledge” (Anstrom et al., 2010, p. iv), which includes new words and a new way of speaking, or register, that students do not encounter until they hear them for the first time in the classroom. This definition is very clear, however, there are alternative definitions shepherding further dissection of academic language properties. An alternative definition of academic language is “the specialized language, both oral and written, of academic settings that facilitates communication and thinking about disciplinary content” (Nagy & Townsend, 2012, p. 92). Using this definition of academic language provided by Nagy & Townsend (2012), the essence of what academic language is can be further dissected into several components.

There are six characteristics of academic language all working in conjunction: Latin & Greek vocabulary, morphology, nouns/adjectives/prepositions, grammatical metaphors and nominalizations, density of information, and abstractness (Nagy & Townsend, 2012). The knowledge of the existence of these six characteristics of academic language is useful for facilitating thinking about what content instruction looks like for students to better support educators teaching the language of their content area by raising their awareness of and conveying the abstractness of academic language (Nagy & Townsend, 2012). To facilitate communication of, and to further dissect the abstractness of academic vocabulary, it can be broken down into three essential types of vocabulary: domain-specific vocabulary, general academic vocabulary and language, and generative vocabulary (Helman et al., 2017).

Domain-specific vocabulary refers to the vocabulary used to represent the concepts and objects of a specific area of content, such as math. These words are usually nouns with precise meaning within the subject that is being taught (Helmen et al., 2017). General academic vocabulary and language include phrases and words that are not specific to any one particular content area and may be used in multiple content areas. These general words and phrases allow students to use domain-specific vocabulary in tandem to build meaningful relationships and connect related concepts within and out of the content area being taught. Generative vocabulary is the understanding of the morphological system of the English language. Knowledge of this morphological system assists students with independently gaining new domain-specific and general vocabulary words (Helmen et al., 2017). The tapestry of academic language characteristics can be seen as the language characteristics identified by Nagy & Townsend (2012) intertwined in the vocabulary types identified by Helmen et al. (2017). Table 4 shows examples of how types of vocabulary and vocabulary characteristics intermingle. The intermingling of vocabulary types and characteristics shown in Table 4 is meant as an example, not an exactitude, as other organizational interpretations could be argued.

Table 4

Vocabulary Types and Characteristics

Domain-Specific Vocabulary	General Academic Vocabulary	Generative Vocabulary
Latin & Greek vocabulary	Latin & Greek vocabulary	Latin & Greek vocabulary
nouns/adjectives/prepositions	abstractness	Morphology
grammatical metaphors and nominalizations	grammatical metaphors and nominalizations	grammatical metaphors and nominalizations
density of information	nouns/adjectives/prepositions	nouns/adjectives/prepositions

Linking Language and Content Learning

There are five language development stages that can classify the level of an EL student (Haas & Brown, 2019). Of those five stages, the first four identify *multiple repetitions of language* as the number one method of how students gain language (Haas & Brown, 2019). Effective vocabulary instruction for EL students includes students receiving multiple opportunities every day for language use, as identified by Haas & Brown (2019) along with Helman et al., (2017). Considering that linking language instruction and content instruction is an effective teaching practice (Schleppegrell, 2007), explicitly teaching domain-specific vocabulary simultaneously with content instruction should be integrated into all instructional activities (Helman, 2008).

A lack of academic language has been found to be the culprit, in part, of the achievement gap that separates EL students’ scores from non-EL students’ scores (Anstrom et al., 2010), specifically scores on standardized tests. Support for linking language and content learning comes from an interpretation of research that points to learning the language of a new discipline or content area and learning the content of the new discipline to be inseparable (Schleppegrell, 2007). Specifically, concerning the discipline of math, domain-specific math vocabulary needs to be taught in the context of problem solving for students to learn how to categorize and express terminology (Helman et al., 2017), and knowledge of math terminology is a predictor of achievement scores on math exams (Ünal et al., 2021).

Intentional Vocabulary Instruction

Strong vocabulary instruction has three characteristics: language and word-rich environments, intentional vocabulary teaching, and teaching students word-learning strategies (Blachowicz et al., 2006). These effective vocabulary instruction characteristics are in line with the notion that language and vocabulary instruction is inseparable (Helman 2008; Schleppegrell, 2007), the essential types of vocabulary knowledge identified by Helman et al. (2017), and the morphology characteristic of academic language identified by Nagy & Townsend (2012).

The review of research for this body of literature has stated a clear definition of academic language, the components and characteristics of academic language as well as the importance of intentional vocabulary instruction. The remainder of this section will focus on the best practices for teaching domain-specific vocabulary and generative vocabulary so that students may become more self-sufficient with future vocabulary acquisition. General academic vocabulary, vocabulary that is used in academics at a higher frequency than nonacademic vocabulary and used across all disciplines (Nagy & Townsend, 2012), will need to be instructed as needed due to its abstract nature (having multiple meanings/grammatical metaphors).

Domain-Specific Vocabulary Instruction

Domain-specific vocabulary is needed to be able to comprehend the subject being taught (Helman et al., 2017). Educators should develop and use explicit and systematic vocabulary instructional plans throughout the entire school year to best support students in learning math vocabulary. However, vocabulary instruction during the teaching of math is often overlooked (Riccomini et al., 2015). Teachers should begin giving domain-specific vocabulary instruction during math class in the form of connecting math to their world (Cummins & Man, 2015; Helman et al., 2017; Riccomini et al., 2015) using anticipation guides, telling stories involving math, and using concept maps to map out characteristics, examples in math curriculum, examples in their own lives, and pictures (Helman et al., 2017; Riccomini et al., 2015) to engage students and build their conceptual knowledge of math (Helman et al., 2017).

The next step in teaching domain-specific vocabulary in math is to find and name patterns and make them explicit to students. Mathematical patterns tie vocabulary and concepts together (Helman et al., 2017). Discussion of these patterns engages students in a dialogue, assisting with language development (Cummins & Man, 2015) and involves the use of common/conversational language to teach students the pattern signal words, verbalization of those patterns, and the ability to visualize them (Helman et al., 2017). This model allows students to use the vocabulary of math in context, enriching their understanding of terms, concepts, and mathematical writing abilities (Helman et al., 2017).

Generative Vocabulary Instruction

The English language mainly consists of Greek, Latin, and Anglo-Saxon prefixes, suffixes, and roots, which can be used to understand the meaning of new words (Ebbers, 2004). The ability to generate words or understand the structure of the English language is known as morphology (Rastle, 2019). By using the domain-specific vocabulary of a lesson as a launchpad for generative vocabulary instruction, students can learn the system of meaning embedded in the English language (Helman et al., 2017). For example, after teaching a lesson on triangles, Helman et al. (2017) identify that the word can be separated into two parts, *tri* and *angle.* After explicit instruction of what each part means, students can generate their own understanding of new words such as *triple, tricycle, triceps,* and *quadrangle, rectangle, pentangle* and so forth using the knowledge gained of prefixes and suffixes.

This third and final body of literature has explained and defined differences between common/conversational English and academic language. Educators must understand the key differences as well as the characteristics and types of vocabulary that comprise the vocabulary of academic language in order to teach it effectively, as it is theorized that learning the new language that specifically correlates with new math concepts is easier for EL students than learning technical words that have multiple meanings outside of math class (Schleppegrell, 2007). Although there are discrepancies between scholarly opinions on the parameters of what constitutes academic language, knowledge and explicit instruction of differing types of academic vocabulary are fundamental to EL students' language acquisition. General academic vocabulary, which is abstract and used across all domains of academics (Nagy & Townsend, 2012), is essential vocabulary for building meaningful relationships with domain-specific vocabulary and content (Helman et al., 2017). Domain-specific vocabulary includes words only found in a certain content area, and generative vocabulary instruction assists students with independently growing their vocabulary by using knowledge of the English language system. More research is needed to determine the impact of scaffolding domain-specific vocabulary in all content areas (Nagy & Townsend, 2012).

Conclusion

Education for EL students and the federal and state laws governing it are relatively new in the grand scheme of public education in the United States. With the EL student population in the public education system growing throughout the country, direction pertaining to what EL education should look like and how it should be implemented is needed. However, based on the current laws on the federal and state levels examined in this literature review, it is evident that there is no clear direction or interpretation of what EL education should be as most of the laws and regulations put the responsibility on individual districts, schools, and teachers.

Exploring the question of how scholars have understood the relationship between language acquisition, understanding number sense, and confidence concerning EL students paves the way for needed research concerning the relationship between math vocabulary and academic achievement pertaining to upper elementary students (Ünal et al., 2021) and domain-specific language (Nagy & Townsend, 2012). While researchers have looked separately at the importance of the strategies for language acquisition, number sense and student's confidence, more research is needed to understand the relationship between these aspects of learning, particularly for EL math students. To assist in this scholarly endeavor, the exploration of how to best teach EL students exposed a great deal of literature speaking to the many misconceptions faced by the EL student population in the public school system with many of those misconceptions resulting from a multitude of missed opportunities. These missed opportunities in educational settings or opportunity gaps result in the achievement gap that currently exists between EL students and non-EL students.

Scaffolding, providing extra time, providing extra opportunities to learn a new concept, modeling new content, and teaching about metacognition to increase EL students' self-efficacy could benefit EL students learning number sense strategies (Bandura, 1997; Haas & Brown, 2019; Lestari & Jailani, 2018; Muhid et al., 2020). Self-efficacy is the belief in one's ability to control one's own thinking, belief in one's self is having confidence, and the more confident EL students are in math (Yang & Lin, 2015) and the English language (Soland & Sandilos, 2020), the higher their achievement as recorded by standardized tests and exams. Current literature does not specify specific measurements of math vocabulary, nor does it explicitly examine domain-specific math vocabulary and the role it may have in math instruction (Ünal et al., 2021). My proposed research will use what is known about academic language and vocabulary, consisting of the various vocabulary characteristics and types, EL teaching strategies, vocabulary teaching strategies, and what is known about competing number sense constructs to explore the current gap in literature pertaining to upper-elementary students learning domain-specific vocabulary to increase their confidence and ability using number sense strategies.

CHAPTER THREE: METHODOLOGY

Introduction

In this chapter, I outline the underlying philosophical assumptions and their implications influencing this qualitative action research study. I begin by presenting the research design, ontology, and research questions. Next, I outline relevant background information surrounding the research site, participants, my role as a researcher, and research ethics about this study. Then, I will outline the information on instrumentation and protocols for data collection and analysis, and I will end with a discussion of possible limitations of the study.

Research Design

Ontology

The design of this qualitative study is action research based using an interpretivist lens to examine EL students' understanding and confidence in number sense strategies, to develop a better understanding of the area being investigated with the intent to implement meaningful change (Stringer & Argón, 2021). The main difference between action research and other forms of research is the cyclical structure of continuously working toward a resolution with participants based on the findings during the study. Hinchey (2008) wrote that the process of action research was being done by teachers every day. However, actually doing the research formalizes the process in a structured way by focusing attention on one area for an extended amount of time (Hinchey, 2008). I worked with my participants using new knowledge acquired through data collection from research to increase their understanding and confidence in the application of number sense strategies.

The term *action research* refers to a group of many different types of research (Hinchey, 2008), and the type I conducted is called *practical action research*, which focuses on improving the practice of education by identifying a classroom problem and working towards identifying as well as implementing improved strategies (Hinchey, 2008).

I had heard the notion that math is a universal language from paraprofessionals, teachers, and administrators on several occasions during my time in education. While the Hindu-Arabic numeral system itself is the most widely used around the world (Danna, 2019), the language that the numbers are embedded in is not. As a child in grade school, math was not always easy, but I was able to understand the main objectives of what I was being asked to solve because math was being taught to me in my first language. When thinking about math instruction with an interpretivist lens, it is apparent that math is anything but universal.

From a bounded relativist ontological assumption that there are multiple realities (Moon & Blackman, 2014), it is obvious that my reality of being taught math in my first language is not the reality that all students experience. Our world is full of these multiple realities that are occurring for multiple people for multiple reasons in all aspects of life, including in the classroom, sometimes without paraprofessionals, teachers, administrators, and students ever noticing their existence.

This research study was designed to add to the conversation concerning the reality that not all students are instructed in math in their first language (creating multiple student realities in the classroom) and to query possible relationships between understanding number sense, math vocabulary, and student confidence in mathematical reasoning through analysis of participant data gathered in an upper-elementary math classroom where I, the teacher, am also the researcher and an insider (Creswell & Poth, 2018).

Research Questions

This study will be guided by the following research questions:

Central question: What role, if any, does math vocabulary instruction have in increasing upper elementary students' confidence in their understanding and application of number sense strategies?

Sub questions:

5) What role does grade-level math vocabulary instruction play in enhancing the confidence level and use of number sense strategies of upper elementary EL students?
6) What role does spiral math vocabulary instruction play in enhancing the confidence level and use of number sense strategies of upper elementary EL students?
7) How do upper-elementary EL students perceive that math vocabulary instruction contributes to their understanding of early number sense strategies?
8) How do upper-elementary EL students perceive that math vocabulary instruction contributes to their understanding of mature number sense strategies?

This study's research question and sub questions were designed in parallel to create an overarching qualitative central question with sub questions to further specify distinct areas of inquiry (Creswell & Poth, 2018). The central question is intentionally open-ended to leave room for conducting cyclical and intentional action research procedures in tandem with participants to provide relevant knowledge concerning number sense and math vocabulary as well as creating meaningful change in the form of an effective outcome to the study (Stringer & Argón, 2021).

Research Site and Participants

The research took place in my math classroom at a school in the state of Minnesota in the U.S. with permission from the school's administration, participants' parents and guardians, and the participants during the 2022-2023 school year. The school serves students in the elementary and middle school grades and had a total enrollment of 445 students at the time of the study. This school was selected for this study because action research requires direct involvement of the researcher and I have access to the students in my classroom to be participants in the study with parents' and participant's approval, and the support of school administration. The student population is mostly EL students; the exact percentage of school wide EL students during the 2022-2023 school year was 39%, however, the EL students' percentage for the participants' grade was 79% when including students who had received EL services in the past. I taught math and science to two sections of upper-elementary students accounting for a total of 29 students, with 23 of those students having a first language other than English and 12 of those students participating in the study. All students in the class, regardless of participation in the study, received the same instruction and classwork during the school year to ensure equal treatment. Each class received at least 70 minutes of math instruction per day, which allowed ample time for the collection of data for this study during the school day.

Since action research is usually focused on a group of individuals, such as a classroom of students, the type of sampling used for this study was purposive sampling of the EL learners in my classroom (Fraenkel, et al., 2019). I have a good relationship with the families at my school as I taught the students who participated in the study during a previous year at a lower grade level when I completed my student teaching. Students' and parents'/guardians' consent was obtained through a signed permission form by both the student and the parent/guardian.

Positionality

In this section I will explain my current understanding of where my positionality lies within my position as a teacher, which is to educate my students; and my position as a researcher, which is to advance the scholarly conversation in my area of study. I will also explain how my personal mission and values will assist my two identities in informing and influencing one another.

My research aligns with the interpretivist paradigm as interpretivists see reality as subjective and constructed (Lather, 2006). I have an ontological view of each student having their own reality. No student sees or experiences the world the same; therefore, no student learns the same. There may be close similarities, but they are not exactly the same. These multiple perspectives and interactions with the world create multiple realities where students construct their own meaning as they interact with the world around them. How EL students interact with math in the general education classroom (when the instruction is in English) is different from students who are native English speakers. Many native English-speaking students are only interacting with new math concepts for the first time, whereas many EL students are interacting with both the new math concepts and new vocabulary in tandem, thus creating multiple realities within the general education classroom for students.

For my positionality as a teacher, my experience teaching EL students for most of my teaching career over the past decade has constructed how I view education. Education is a cyclical process, for teachers and students and there is always room for improvement. For many educational entities around the world, and especially in the United States, the focus is not on improvement, but rather on student performance on standardized tests; measuring how students perform compared to other students within a single reality.

My position on testing, unlike the United States, is to focus on student growth, rather than proficiency, as proficiency is an unfair measure of knowledge as well as unobtainable without growth. By continually striving for evolution of self, I lean on my philosophy of continuous improvement. School should be an enjoyable learning environment for all students. The students whom I teach love learning but are discouraged and disappointed by standardized tests. By focusing on and celebrating each student's progress instead of a measurement of how each student compares to the masses, educators can be more informed of how to advance each student in their learning endeavors. Unfortunately, standardized tests are not going anywhere anytime soon. Since standardized tests are a gateway for acceptance into higher education programs, they are more of a barrier for students who are unable to demonstrate their knowledge in test form. This reality of standardized testing being here to stay is a monumental reason why students need to be taught how to fit into a reality they are not familiar with or confident in and language acquisition may be the answer.

My research is based on exploring how upper elementary EL students best learn number sense. I worked diligently to ensure my assumptions did not influence my research, and ensured that I separated my roles of teacher, researcher, and colleague by following strict ethical practices. I kept my own known and unknown assumptions from influencing my research by focusing on keeping my research ethical when studying underaged participants, understanding my own positionality, being reflexive, leaning into action research principles and practices, and keeping myself grounded in my mission and values.

Research Ethics

For researching elementary students, I obtained written consent from each parent or guardian for each student involved in the study. This consent was translated into both Hmong and Karen languages by my Hmong and Karen colleagues. I informed my students and their parents or guardians that there was no requirement to participate. I discussed the study with my students and their parents or guardians detailing how the study was integrated into regular classroom instruction and how it was connected to each student's learning. If they did not wish to participate, it had no bearing on the students' grade in my class. All students participated in the same classwork and activities that were used as formative assessment data for the study. No student did anything different than any other student in the classroom, regardless of participating in the study or not. There was no control group as it would create an ethical dilemma.

The formative assessments and classwork for this study were not graded. The assignments used for grading were the summative assessments provided by the school's curriculum, which were used to show if there was student progress. The data collected for the study were saved in a secure way to protect the personal information of the students; my school's secure computer drive, where the rest of our student information was kept, was a suitable location for classwork material. Other paper documents were not stored on site but rather in a secure location in my residence. Data pertaining solely to students participating in the study were uploaded to a secure coding website called ATLAS.ti. When students returned the form, I made a copy of each form and gave it back to the students to bring back to their parents or guardians for their record keeping.

I also protected my participants to ensure they did not feel overwhelmed or feel they were having their education sacrificed for my study. My strategy for this was to integrate my study into the regular coursework for the entire class in a cyclical manner to support meaningful change for the students and their families to align with action research practices (Stringer & Argón, 2021). All the students in my class were engaged with the same lessons, content, and assignments. However, only data from willing participants were collected and used for the study.

I also had strategies in place to protect student's emotions when it came to completing classwork that was used for the study. I implemented strategies to collect data in a way that also allowed students multiple attempts to complete a problem while still receiving full credit for its completion. This is a strategy that I have been using for my entire teaching career. My goal as a teacher is to ensure students learn the content that I present to them. Some students understand it right away and get a perfect score, however most students take three, four, or more tries to get it right. I always tell my students that “it is okay not to know something, that is the reason why they are in school.” It is important to provide the students with an appropriate grade and/or reward for completion, regardless of how long it takes.

Data Collection

In qualitative research, there are three techniques to collect data that most researchers utilize while conducting a study (Fraenkel, Wallen, & Hyun, 2019): observing participants in their natural environment, interviewing participants to gain insights about opinions, ideas, and experiences, and collecting documents to be analyzed. Not all qualitative research utilizes all three techniques, however, for my study the triangulation of multiple data points was used to gather information pertaining to different aspects (Maxwell, 2013) of number sense and to increase the trustworthiness of the study. This approach to broadening the range of aspects addressed during the study, rather than focusing on one specific conclusion, blended well with the intent of action research to find a practical solution to the phenomenon being investigated (Stringer & Argón, 2021). While triangulation of data is often used for quantitative and mixed methods research, it is also used in social research to reach data saturation (Fusch & Ness, 2015). Data in the form of documents, such as student journals and test scores, along with observations, and interviews were triangulated to form and support themes that emerged from the study.

One cycle of data collection, or one “action cycle,” was, on average, a two-week period from when a new topic was introduced, to when students were given a test from the provided curriculum. However, to address number sense strategies and vocabulary from prior years that were missing from many students, action cycles did not always align with the introduction of new topics to allow for unplanned spiral lessons. Some action cycles were three weeks in length due to extenuating circumstances, such as snow days, testing days, or distance learning that would have resulted in insufficient data if the action cycle was not lengthened. Several types of data were collected during each action cycle (Table 5). Students were given a math journal entry two to three times per week on average. Data from observations were collected daily in the form of systematic anecdotal notes. As action cycles began in January 2023, there were eight data cycles during the 2022-2023 school year along with nine weeks of classroom observations in the form of systematic anecdotal notes starting in October 2022. Action cycles and data collection other than classroom observations did not start until January 2023 as clearance for the study was granted by the Concordia University St. Paul Internal Review Board (CSP IRB) in December 2022.

Data from observations were integrated into action cycles beginning in January 2023 when school resumed after winter break. For the first part of the year up to December, I recorded observations every day. No action memos, student journals, or student interviews took place until after we returned to school from winter break. Student interviews were conducted with each student once before and once after the state standardized math exam in mid-April. Each student was asked a total of 11 questions between both interviews (Figure 1). The aim of these interviews was to collect data from the student perspective on math word problems to triangulate data from observations, action memos, and student journals.

Figure 1

Interview Protocols

Pre-Standardized Test Interview Questions

1) What are your feelings when you read a math word problem? Why?
2) When you read a math word problem, how confident are you that you will understand what you are asked to do?
3) What, if anything, is difficult about understanding a math word problem?
4) Other than the words in a math word problem, what, if anything, is difficult to understand?
5) What do you do when you do not understand a word in a math problem?
6) What do you visualize when reading a math word problem?
7) What, if anything, helps you visualize a math word problem?

Post-Standardized Test Interview Questions

1. In January, you said you felt "________" when you read a math word problem. What is your feeling now? Do you still have that same reaction?
2. How confident are you now, when reading a math word problem, that you will understand what you are asked to do?
3. What is difficult about understanding a math word problem now?
4. Do you use any of the strategies that we practiced in class when you read a word problem? If yes, which ones and how? Please give me an example.

Table 5

Overview of Data Collection

Data Type	**Collection Schedule**	**Purpose**
Two Week Action Cycle Data		
Student Math Journals	2-3 times per week	• Check for student understanding and use of number sense strategies • Check for student identification of and understanding of academic vocabulary
Vocabulary quiz	Once a week (for only 4 weeks)	Check for student vocabulary understanding
systematic anecdotal notes	Daily	Capture observations of student understanding or growth
Biweekly math tests	Every two weeks	Monitor student performance on curricular assessments
In-class assignments (worksheets/workbook)	Daily	Monitor student understanding of current standards and number sense
Additional data		
Individual Student Interviews	Twice – at beginning and end of study	To involve stakeholders
MAP Test Data	Twice a year. Once at the beginning of the school year, once at the end of the year.	Measure student growth

One area of analysis was levels of student understanding of different types of academic vocabulary. A math vocabulary quiz was given once a week to collect data on student understanding of different types of academic vocabulary. There were only four vocabulary quizzes given before I changed my vocabulary acquisition strategies in the classroom due to many students struggling to learn math vocabulary from this teaching method. Observations from student math journals were recorded and analyzed in action memos. Data from student math journals were coded and analyzed for themes after all data were collected at the end of the school year. Information from observation of student math journal participation was used to inform vocabulary instruction to enhance student confidence levels in and use of number sense strategies. Aside from the ongoing classroom data, I used standardized test data along with student interview data. These each occurred twice: once near the beginning of the study and once near the end of the study.

Documents

Documents were the main source of data. For documents, I analyzed student journals, tests, and worksheet scores, as well as data from national exams such as the MAP test, which is possible to use with consent from the school. The main document used as data for the study were math journals that used a modified approach from a study conducted by Yang & Lin (2015), which tasked students with the following:

1) Answering a math question from a specific area of number sense.
2) Identifying their confidence level in their answer to the question.
3) Identifying the number sense method used to solve the problem.
4) Identifying their confidence with their understanding of the number sense method they used to solve the problem.

I implemented a similar design as described by Yang & Lin (2015) in which a 4-tier diagnostics test (listed above) was given to 195 fifth grade students in Taiwan. The study that I implemented in my classroom measured these tiers, but also included two more questions about domain-specific math vocabulary and confidence levels related to the student initial math vocabulary knowledge. These two additional questions were added after the first round of journals: "What did you visualize when reading the math word problem?", and "What, if anything, helped you visualize when reading the math word problem?" Students were asked to identify a vocabulary word, or words, in the math problem and explain what that word (or words) meant in relation to number sense strategies. Then, the students identified their confidence level with their understanding of the word they chose to identify and explain.

Yang & Lin (2015) focused on studying five broad areas of number sense. For my study, I planned to cover the eight areas of number sense identified by Andrews & Sayers (2015), which were labeled as foundational number sense (FONS). FONS has parts of both early number sense (ENS) and mature number sense (MNS). However, when I was creating word problems for student journal entries during the school year, it was counterproductive to create word problems based on specific number sense components. The more practical way of creating the word problems was to first base them off current grade-level standards and then identify the number sense components and vocabulary that correlated with those grade-level standards. This led to the creation of Appendix A & Appendix B, which describe what number sense components are embedded in each grade-level standard in K-5 and at what grade level number sense components are introduced. Therefore, I covered ENS, MNS, FONS, one ANS component, and eight refined number sense components that I identified after coding the data from the study. Creswell & Poth (2018) describe participant journals as being a data collection approach for qualitative research. Instead of creating a lengthy exam for students, I requested that the participants keep a math journal, which tracked math vocabulary and math strategies related to word problems and the number sense components within them, along with the participants' understanding of and confidence levels for problems they are tasked with solving during math class.

In addition to the math journals, I monitored and analyzed the vocabulary quizzes I assigned as well as the math tests, worksheets, and other assignments that were part of the set curriculum. Our school uses *Reveal*, a math curriculum developed and distributed by McGraw Hill. The MAP test is unlike a standardized test, as it measures student growth rather than proficiency. After students complete the first MAP test, a goal is generated based on their individual score. Data from the first and second MAP test will be analyzed for student growth over the course of the school year.

Interviews

For interviewing participants, I interviewed each of my math students individually and only used the data from students participating in the study. I created a flexible schedule that allowed for interviewing each student twice during the school year; at the beginning of the study and near the end of the study, which aligned with the end of the school year. This schedule did not need to be integrated with regular math instruction time as the morning class period was an advisory period, which was a suitable time to use for student interviews. Both students participating in the study and students not participating in the study were given the same assignments. The only difference being one group of student data was collected and analyzed for the study while the other student data was not. The reason all students were interviewed was to improve teaching practices so that students may gain a better understanding of number sense through vocabulary instruction. Thus, student feedback was used to modify classroom instruction, a practice that I have been implementing each school year.

When designing the study's interview conversational guide, the structure of the interviews was guided to minimize extra stress and anxiety on participants and myself (Rubin & Rubin, 2012). One way to decrease anxiety was to be prepared for the type of interview being conducted. Since I needed to have shorter interviews, a brief interview pattern referred to as *picking up the twigs* (2012) was useful as this involved a continuous conversation with the same participant over the course of several short interviews. Only two interviews were conducted as a method to gather information, but not take away from instructional time. To gain data saturation through triangulation, *picking up the twigs* (2012) allowed me to bring back the first interview and tie the study together with the second and final interview.

Observations

The form of observation I chose to fully participate in all activities and be transparent of my role as both teacher and researcher was *participant-as-observer* (Fraenkel, Wallen, & Hyun, 2019). Since I was already participating as the teacher in the setting that I observed, this role of observer was appropriate for me in my study. I was facilitating the learning in the classroom, so I was not always doing exactly what the students were doing, but I was still fully participating in the classroom activities.

Observations were an integral part of the design of my study to reach data saturation. I collected data in the form of systematic anecdotal notes in a notebook from observations after teaching whole-group instruction. I did this by walking around the room and observing how many of the participating students were able to correctly show their knowledge by completing math problems using number sense and vocabulary strategies. Systematic anecdotal notes have no official format to be followed other than recording specific and concrete descriptions and the date it happened (Fraenkel, Wallen, & Hyun, 2019), and it was responsibility of the observer to decide what is important and in what way to record the observations. Observations of students in their natural environment took place during individual practice and group work where members were discussing math word problems and solving the math word problems together using number sense and vocabulary strategies. I paid attention to participants' use of terminology that was taught during the lesson along with looking for a detailed description of the math word problem solving process. These were observations I normally do as the teacher, regardless of there being a study in place or not. In the classroom, systemic anecdotal notes refer to the daily written recording of my observations. The systematic anecdotal notes provided me with data relative to the main stake holding group (the students) in their everyday setting, where they performed the tasks being studied (Stringer & Argón, 2021). In other words, this data was there already, it just needed to be recorded and used for data saturation via data triangulation to extend the pool of information available to the study (Stringer & Argón, 2021).

Preparation and Data Analysis

Preparation for document collection was crucial to the outcome of this study. Notebooks for math journals needed to be purchased and passed out to all students regardless of if they were participating in the study or not. Expectations for the use of the journal needed to be clarified regularly to ensure the participants were completing their classwork properly. Per the Internal Review Board (IRB) process, interview questions for the study needed to be created ahead of time for participants. The day prior to being given to students, I created original math word problems about specific areas of number sense using specific vocabulary that also fulfilled Minnesota math standards. A confidence level question for their answer, a reason for their choice based on understanding of math vocabulary used, and confidence level for their choice were also given to students for each math journal entry. For the first round of interviews, students were recorded on an audio recording device to save their discussion for later transcription. The audio was deleted after transcription. For the second round of interviews, an audio recording device was not used as interviews were transcribed immediately as questions were answered. Interviews took place outside of the action cycle of every two weeks as students were only interviewed twice. During the action cycles, any changes to instruction were recorded via memos and observations of those changes were also recorded. The memos concerning changes to instruction and all data collected during each two-week cycle were integrated into the action memos.

Creswell & Poth (2018) discussed coding qualitative data to identify themes. This data analysis strategy was helpful when identifying main themes of what students say was difficult for them concerning number sense strategies. Researchers must pick one open coding category to build toward the central phenomenon in the process. After, researchers create categories around the identified central phenomenon (Creswell & Poth, 2018). In action research, analysis of data follows the steps of reviewing: unitizing (combining), categorizing and coding, identifying themes, creating a category system, and developing a framework for report (Stringer & Argón, 2021).

Data from my study points to math vocabulary as the central phenomenon. I followed the steps listed by Creswell & Poth (2018) and Stringer & Argón (2021) by first reviewing my data by reading through all observations, action cycles, journals, and interviews from my study and then uploading my data by hand into ATLAS.ti, a coding software used for qualitative data analysis. I then combined my data in ATLAS.ti to categorize and code that lead to the identification of themes throughout the school year. These themes allowed me to see the connection between student work, student responses in journal entries, student responses to interview questions, and student behaviors and interactions in the classroom.

Limitations

The limitations for this study include the purposive sampling method of choosing participants in my own classroom that were EL students. The timeframe for this study was nearly an entire year of school, from late October 2022 to late May 2023. After IRB approval, all documents, observations, and interviews were conducted and collected during this timeframe, except the MAP test, which was taken by students at the beginning of the school year. Observations were recorded prior to IRB approval, but only used for the study once approval was granted. Once consent had been granted for student data collection by participants and their parent or guardian, MAP data and classroom data for the study were collected for analysis. Although several months during one school year was a sufficient length of time for a dissertation study, a longitudinal study of several years would be better suited to examine the effects of domain-specific and other academic vocabulary instruction as students progress from kindergarten through elementary school.

Conclusion

In this chapter, I explained the research design of action research and the specific type of action research I conducted, which is *practical action research,* and how this type of research aligns with my positionality and ontological views, being bounded relativism view of multiple realities existing, namely for upper-elementary EL students in my math classroom during the 2022-2023 school year. The research questions were designed to work with the action research formula of conducting cyclical research in the classroom by an insider. Ethical considerations were taken into account in the design of the research to prevent any unintended segregation or groupings of students. The cyclical nature of action research requires a research cycle schedule to be set, which was every two weeks during November to May. Interviews were conducted outside of this two-week cycle. Documents and observations served as the data for changes to instruction during the action research process. Analysis of documents, observations, and interviews took place after the 2022-2023 school year to determine if the research question and sub questions were answered.

CHAPTER FOUR: FINDINGS

This study started with a goal to better understand the relationship between the English language and math, leading to the question: *How have scholars understood the relationship between language acquisition, number sense ability, and confidence concerning EL students?* The answers to that question were presented in Chapter 2 and will be meshed with the answers to the central and sub research questions.

I will first present data from this study in chronological order, that is, in the order that themes emerged through classroom observations and action memos during the school year. Non-sequential data from student journals and interviews will be woven into sequential observation data to identify the themes, with the main themes that arose during that time frame as the header of each section. Next, I will present how the themes in the data can be used to answer the sub questions to the central research question. Finally, the themes in the data will be used to create a comprehensive response to the central question of the study. Discussing the central question last serves the purpose of combining all themes in the data to create a conclusion for the data and themes in the study. Where this study fits into the scholarly conversation will be discussed throughout Chapter 4 to make recommendations for policy, leadership, instruction, and further research concerning upper elementary EL students learning in the math classroom during whole-group instruction in Chapter 5.

Themes from the data

Since my research study had the aim of improving math vocabulary for EL students, I began the school year giving students various tasks to improve their math vocabulary. During the school year, those tasks evolved based on my classroom observations and action memos of the students' performance in class, including students' feedback elicited from whole-class discussions.

Math Vocabulary & Structured Dialogues

From October to December, students were taught vocabulary using two main methods. The first method required students to record definitions of domain-specific vocabulary from the math curriculum glossary along with a math example and use the word in a sentence, as depicted in Figure 2. Only the domain specific words were introduced and discussed, as I expected the students to be able to create their own sentences using general academic vocabulary.

Figure 2

Example of Domain-Specific Vocabulary Worksheet

NAME:
Directions: Please define the word, use the word in a sentence, and show a math example.

Definition	**Use in a Sentence**	**Math Example**
Multiplication **Example:** Repeated addition	**Example:** Six multiplied by five is the same as adding six 5 times.	6x5=30 6+6+6+6+6=30
Reciprocal		
Equivalent Ratio		
Divisor		
Dividend		

The second method was to give students a structured dialogue to read back and forth with a partner or small group during math class. I had been taught how to implement structured dialogues in lessons during a professional development session a few years ago. I had used them a few times over the years in different classroom settings and had success with students learning new vocabulary and content. The structured dialogues I created for the study took the form of call and response, where I would write and say a sentence followed by students repeating it back to me or providing students with a pre-written dialogue that I created, like the example in Figure 3.

Figure 3

Structured Dialogue Example

Have a discussion with your neighbor about variables!
Student A: Can you help me understand how to graph sets of equivalent ratios?
Student B: Yes, I can! Equivalent ratios show the same relationship between two variables.
A: What are variables again?
B: A variable is a symbol, usually a letter of the alphabet, that holds the place of an unknown number.
A: So, the number of students present in X grade today could be represented by the symbol, or letter, ***"s"***, because we do not know how many students will be in school today until after all the buses get here.
B: Correct! And you need to understand that there are TWO different types of variables: **independent** variables and **dependent** variables.
A: I remember that an **independent** variable is what a researcher is in control of.
B: Yes! And **dependent** variables are what the researcher **measures** or **counts**.
A: I am feeling confident about knowing the difference between independent and dependent variables now, but I still do not understand how to graph them.
B: Do not worry, I will help you! Remember, an ordered pair includes two variables (x, y). They are called the x and y coordinates because we use the x-axis and the y-axis to graph them on the coordinate plane.
A: So, is the (x, y) ordered pair is replaced by the independent and dependent variables?
B: Yes they are! The independent variable is ALWAYS the x-coordinate and the dependent variable is ALWAYS the y-coordinate.
A: If we were counting the number of shoes worn by a group of students, we would first need to know how many students were in the group.
B: Exactly. Since we are in control of how many students are in the group, the number of students would be the **independent** variable.
A: And the number of shoes would be the **dependent** variable, since we are counting them!

B: Show me what you learned and graph the amount of shoes worn by groups of 3, 6, and 9 students! Don't forget to label the x and y axis with the correct variables!

After coding the first few months of observations, a discovery surfaced: students struggled to learn math vocabulary through searching and writing vocabulary definitions. During November, I observed that, "Students were challenged with finding the definitions on their own in groups — it did not go well. Even after several days of working on vocabulary, many students were not able to identify or use domain-specific or general academic vocabulary in math word problems or in discussion." During December, I observed that:

> "Many EL students struggled to use the math vocabulary in a sentence that made sense. A lot of time was spent during class, over 30 minutes per class, as many students needed extra time to learn vocabulary this way, and usually many students were still not finished when it was time to move on to other tasks. The extra work was assigned as homework; however, many students were unable to complete the work without assistance."

Giving students a structured dialogue to read with their classmates was a much more successful method based on observations in the classroom. The structured dialogue consisted of spiral vocabulary, domain-specific, and general academic vocabulary (including synonyms) in sentence form to explain how to complete the steps of various word problems. (The term, "math vocabulary", will now be used to refer to all three forms of vocabulary: general academic, domain-specific, and spiral.) I observed that students could discuss math word problems with their classmates with greater ease after practicing with a structured dialogue. Students who used math vocabulary during group work had greater retention of the new vocabulary throughout the week.

The decision to switch to structured dialogues was based on observations, vocabulary quiz scores, and student feedback. Structured dialogues took a lot less time than vocabulary worksheets and gave the students the structure they needed at a slower pace that they asked for. During December, I noted, “students asked for a slower-paced teaching of the new vocabulary words.” The time saved by giving the students definitions and examples in a dialogue I created for them was reallocated to allow students to spend more time using math vocabulary in context instead of searching for the words in the back of the book and writing down the definitions themselves.

Beginning in January 2023, observations were turned into action memos and students began writing in their student journals. These action memos included my observations of the students during class as well as in their homework, main themes that emerged from student journal entries, and student interactions during work time. The first few rounds of action memos revealed again that students struggled learning math vocabulary when learning from studying definitions and writing their own math sentences. These results are similar to the findings of a critical review by Ellis (2016) on studies concerning task-based language instruction known as focus on form (FonF). Tasking students with studying definitions and writing their own math sentences is similar to a FonF type called text-enhancement. Studies on text enhancement have found that less proficient EL students struggle when tasked with simultaneous comprehension and attention to linguistic form (Ellis, 2016). Vocabulary worksheets required an extreme amount of instructional time; therefore, I decided to shift completely away from vocabulary worksheets based on observations in the classroom and current research.

My observations of students using worksheets for vocabulary acquisition echo the findings that EL students have difficulty showing what they know by manipulating language to represent their thinking with academic concepts (Haas & Brown, 2019); in this case, number sense ability. Many EL students are not able to manipulate language to represent their thinking using academic language (Cummins & Man, 2007). If EL students are not explicitly taught academic language, then they may be perceived as less educated and incapable of performing an assigned academic task by their teacher or school (Barwell et al., 2015; Shleppegrell, 2012). Teaching multiple types of academic vocabulary simultaneously matches current research that shows general academic vocabulary helps students use domain-specific vocabulary in tandem with general academic vocabulary, which builds meaningful relationships and connects related concepts within and out of the content area being taught (Helmen et al., 2017). The structured dialogues I created are a way to scaffold math vocabulary (academic language) for EL students to show their thinking with number sense ability (academic concepts). The structured dialogues I created are similar to two different types of FonF: pre-task planning and task-repetition (Ellis, 2016). The structured dialogues I created for my class were pre-planned, and pre-planned tasks in a teaching context have a positive overall effect on fluency (Ellis, 2016). Task repetition, however, had some mixed results across studies but could also lead to increased complexity, accuracy, and fluency of the English language (Ellis, 2016). While FonF language instruction has similarities to the language instruction in my classroom, it should be noted that it is unclear how many of the studies used for the Ellis critical review (2016) took place exclusively in the math classroom.

Increased student talk was accomplished through whole-group instruction of each vocabulary word. Student talk has been found to be important when studied in a language arts classroom setting, as students who engaged in talking and arguing more often had better results on exams (Sedova et al., 2019). I am unaware of any student talk studies done exclusively in a math classroom setting. A study on student talk in math classrooms will be suggested for further research in Chapter 5. Call and response of the words and how to use them in a sentence were combined with structured dialogues for students to use and read to each other. My observations showed that students greatly increased their use of math vocabulary in unstructured settings:

> “Students are using vocabulary words more confidently, explaining how to solve math problems, and math word problems, to myself and peers. The increased ability to be able to explain how to solve math problems may have a connection to students increasingly showing their work, and showing the steps… The use of math vocabulary, and the depth of discussion being observed was a big jump from the last action cycle. Students were debating, in a good way, about how to find the answer. If students said something incorrect to the group, another group member would explain why they thought the idea as incorrect, and explain what they thought was correct instead, using math vocabulary. It was a great sight to see.”

The conversations I observed the students having were similar to another FonF type called corrective feedback, specifically, learner-generated corrective feedback (Ellis, 2016). Learner-generated feedback is where students correct each other while working on a task (Ellis, 2016). I observed that these conversations also support research showing peer dialogue leading to cognitive growth through the expressing of contrasting opinions in the pursuit of common goals (Howe, 2010). Students who participated more frequently in corrective feedback showed significant improvement on oral exams focused on conditionals, and there is explicit evidence showing that corrective feedback leads to acquisition (Ellis, 2016). Along with acquisition, EL student comprehension of word problems has been found to improve when students re given the opportunity to discuss the problem with their classmates (MacDonald & Banes, 2017). This supporting research correlates with my study revealing that learning vocabulary through structured dialogues with peers increases student understanding of number sense components.

My observations also revealed that, to the students, spiral math vocabulary is very similar to the new vocabulary as many students do not remember being taught the words during previous years. This could be the result of students learning at home during the COVID-19 pandemic, as I observed during the fourth action cycle that students had a difficult time during distance learning, suggesting that vocabulary introduced during the pandemic while students were distance learning may not have been retained by students as well as it would have been if students were learning in person.

During the first week of the fourth action memo, there was only one day of in-person learning and three days of online learning due to a large winter storm. Students struggled greatly with learning math vocabulary and math concepts during this week. Online learning at my school consisted of 45-minute classes where many students were not engaged due to several external factors, taking care of younger siblings, as an example. This particular week is being specifically mentioned as distance learning during the pandemic was discussed in Chapter 1, pointing out how Minnesota EL students had greater learning loss during the year distance learning took place compared non-EL students (Shockman, 2022). The lessons for the entire week were retaught when the students returned to in-person learning. Difficulties during one week of distance learning shows the importance of spiral teaching of math vocabulary and number sense strategies. Reteaching specific vocabulary and number sense strategies from previous years that support current lessons could benefit students as they may not have been taught certain concepts in person.

Based on the finding that students struggle with spiral vocabulary and how students learn vocabulary best by using structured dialogues, vocabulary instruction was fully integrated into the whole-group lessons by tasking students to explain each step of the problem they were working on. Structured dialogues (See Figure 3) that I created were used at the beginning of class using domain-specific, spiral, and general academic vocabulary in context.

Key instructional suggestions noted for the theme of vocabulary and structured dialogues were not using vocabulary worksheets as a main method of vocabulary acquisition, providing structured dialogues for each lesson that include all types of math vocabulary (domain-specific, general academic, spiral, and synonyms), and allowing students to practice their new vocabulary in unstructured group settings so that students may participate in student talk and learner-generated corrective feedback.

Vocabulary & Visualization

While working with structured dialogues, I noticed that many students were struggling with individual work but were doing fine in groups using a structured dialogue earlier in class or during a previous day. After a whole-class discussion, it was revealed that many EL students did not have a dialogue in their mind to help with visualization. Upon realizing that many EL students were not able to visualize math problems using an internal dialogue (a conversation with themselves in their mind), I provided direct instruction on how to visualize a conversation and a math problem in their mind. Below is an excerpt from an action memo in February 2023 describing why the switch to structured dialogues was made permanently and why teaching metacognition is important:

> "As observed during the last action cycle, there is still a gap between knowledge of the words and use of number sense strategies for many students. Changing the way domain-specific words are introduced to students has helped from what I observed during class. Instead of tasking students with finding and practicing the definitions of given words, like what is done during language arts classes, science classes, and the like; giving the

definitions and sentences directly to the students and having the students practice reading it seems to be the correct method to be used going forward. This, along with further understanding how their brains work (metacognition), will hopefully show more understanding of number sense strategies during the next action cycle, as it has started to show this week during classroom observation."

Students were explicitly taught how to visualize math word problems with regard to current metacognition research. Students were taught about how their own minds work, how to think about numbers using pictures or real-life examples, and how to have a conversation with themselves in their own mind. Current research shows that the central metacognitive strategy is self-explanation, and it requires students to use related metacognitive strategies to describe the mathematical strategies used to solve a given math problem (Lestari & Jailani, 2018), which matched the findings of Yang & Lin (2015). Teaching students how to visualize images and how to create a dialogue in their mind resulted in many students expressing that the vocabulary was extremely helpful in visualizing the problem during their interviews and in their math journals. For example, one student answered in their math journal when asked what they visualized when reading the math word problem:

> "I thought of adding all values together excluding the outlier, then dividing the sum by the amount of cupcakes. I also imagined the 6 cupcakes but instead of sprinkles they had numbers."

The same student who commented about visualizing the sprinkles turning into numbers stated, "I just sometimes, like the numbers pop up in my head and I do what they ask me to do", when asked what they visualized when reading a math word problem. When the same student was asked what helped them visualize, they answered, "The vocabulary or the questions."

Other students gave similar answers when asked about what they visualized and what helped them visualize a math word problem. When asked what they visualized when reading a math word problem their responses included: “It tells me something or shows me a picture in my head”, “I don’t know, the numbers and what it is telling me to do with the numbers”, “I visualize, probably what it means or what it is”, and “Figuring out how to get to the last step.” When asked what helped them, all the students whose examples were used in this paragraph mentioned the vocabulary words introduced and studied in class.

Other than vocabulary being helpful, another theme that surfaced from student interviews was multi step problems being difficult to understand. After using structured dialogues, students would work on math problems and math word problems by showing and explaining each step. During classwork, I observed that students struggle with identifying each step, explaining each step, and not being confident that they completed all the steps of the problem.

A few students also mentioned that vocabulary and the stories (people, places, things) in math word problems assisted them in visualizing solving the math word problem. A few students specifically mentioned in their journals that the vocabulary in the math word problems helped them visualize the math they were asked to do. Other students described how they visualized each word problem without specifically mentioning vocabulary words, but still mentioning, “what was said,” or, “the people in the story,” helping them picture the math problem better.

The key instructional strategies that I found to work for the theme of vocabulary and visualization include specifically teaching EL students how to have a conversation in their mind to help them visualize a problem, along with other metacognition strategies mentioned in Chapter 3. Teaching students how to break down a problem into steps is also an important instructional strategy for teachers to continue to focus on in the math classroom, along with tasking students to verbally explain those steps to teachers, classmates, and themselves to practice having internal dialogues to aid internal visualization of math word problems.

Multi-Step Word Problems & Confidence Levels

The struggle to identify and explain each step in a word problem was also revealed in the data from the students' interviews and journals, as multi-step questions were difficult for most students (75%). According to students, what made these types of questions difficult was remembering all the steps, understanding the ways the questions were worded, knowing the vocabulary words in the problem itself, and how long the question was: the longer the question, the more difficult it was to find the answer (Figure 6). Nine out of the 12 students participating in this study specifically mentioned that multi-step word problems were difficult for them. Six students mentioned multi-step word problems as difficult during the pre-standardized test interviews, and four students mentioned that multi-step word problems were difficult during the post-standardized test interviews. Only one student mentioned that multi-step questions were difficult during both pre- and post-interviews: the student whose journal entry and interview were used as an example for Figure 5 & 6. The reason some students mentioned multi-step problems only during the second interview may be due to the intensity that word problems were focused on at the end of the school year.

Data from this study showed that a student's ability to explain how to solve a problem was significantly correlated with correctly answering a math word problem. Based on the students' journal entries, students who were able to explain how to solve a problem could describe the steps needed to solve the problem and use domain specific and general academic vocabulary in context to describe the number sense strategy they used. Not only were these students able to correctly use the vocabulary given to them in the problem, but they were also able to recall words from previous instruction and/or words that connect with the problem in their explanations (Figures 4 & 5).

Figure 4

Example of a Math Word Problem and Student Response in a Journal Entry

Question 9
Stacy created a cube during math class from a web she cut out of construction paper. If the volume of that cube is 125 cubic inches, what is the surface area of that cube? Use what you know about the properties of cubes to help you solve this problem.
Answer
Correct, showed the steps
Confidence Rating for Answer
Confident
Reason or Strategy for your answer
"I first had to figure out the numbers in the surface area, each side was 5 inches. I just needed to do LxWx6"
Confidence Rating for Reason
Confident
Vocabulary
What word in the question helped you figure out this math problem?
What is the definition of this word?
"Surface area – outside of a shape"
Confidence Rating for Vocabulary
Not confident
Journal Entry
"I could visualize a cube with equal sides, but I needed to figure out each side of the cube to answer my problem of my math equation, I was able to understand more. The word CUBE did help since I know what 3D shapes look like. Cubes also have equal sides, so each side was the same number."

However, the ability to get the correct answer, know and use domain specific and general vocabulary, and correctly explain each step of the problem showing number sense ability does not always correlate to higher confidence in the students (For example in Figure 5). Many students who were able to give correct answers while explaining each step of the problem using math vocabulary either reported they were not confident in their answers/explanations or reported that they were just guessing in their journal entry answers.

Figure 5

Second Example of a Math Word Problem and Student Response in a Journal Entry

Question #15c
Tony is driving to Chicago and had planned to spend $3.25 per gallon of gas. However, the price of gas has increased by 20% per gallon. If Tony's car gets 25 miles per gallon, and he plans to drive a total of 800 miles, how much money will Tony spend on gas?
Answer:
correct answer, showed all the steps
Confidence Rating for Answer
Not confident
Strategy For Answer
"The first thing I did was I multiplied 3.25 with 1.2 and got 3.90. The next thing I did was I divided 800 by 25 and got 32, which I then multiplied it by 3.90 that gave me 124.80 as my final answer."
Confidence Rating for Strategy
Just guessing
Vocabulary

1. What word(s) in the question helped you figure out this math problem?
2. What is the definition of this word?

"The word increased helped me because I know that the price of something has gone up."
Confidence Rating for Vocabulary
Just guessing.

From the students' perspective, even though they correctly identified and explained all the steps and vocabulary, they were still unsure if they misunderstood or missed one or more steps to the problem. The same student whose journal entry was used in Figure 5, mentioned in their interviews (Figure 6) that longer word problems, having large vocabulary words in a word problem, and the possibility of forgetting a step in longer problems were difficult. This possibility of forgetting a step in a math word problem leads to lower confidence ratings in their answer, strategy, and vocabulary as shown in Figure 5. This same student, who showed lower confidence in multi-step word problems even though they showed correct work and got correct answers was a student who actively participated in unstructured dialogue during class, used vocabulary correctly to explain the steps and showed a 14-point growth on the MAP test, which was nearly double the national average of eight during the course of the school year.

Student journals, like the entry in Figure 5, were based on a previous study by Yang & Lin (2015) that implemented a four-tier diagnostic test to study the student's number sense, confidence levels, and misconceptions. A notable result of the Yang and Lin study (2015) was that students had a significantly higher confidence rating for their answer than for the reason they chose the answer. This showed that students may understand how to calculate a problem, but do not understand the mathematical rules about why those calculations produce a correct answer (Yang & Lin, 2015). In contrast, the data for my study showed that out of the 243 journal entries collected from 12 EL students where all confidence ratings were recorded, 80% had the same confidence rating for both the answer and reason (or inconclusive because it was blank), 11% had a higher confidence rating for the answer than the reason, and 9% of the students' journal entries had a higher confidence rating for the reason than the answer. The data for my study shows a high correlation between confidence rating for answer, reason, and vocabulary. Meaning, if a student was confident in their answer, they were more likely to be confident in their reason as well as their knowledge of the vocabulary in the word problem. Also, if a student was *not* confident in their answer, they were more likely to be *not* confident in their reason as well as their knowledge of the vocabulary in the word problem.

Figure 6

Example of Student Responses to Interview Questions

Pre-Standardized Math Test Interview Questions

What are your feelings when you read a math word problem? Why?

"If the wording is hard, like, hard words that include a lot of numbers and stuff it is hard for me, but if it is really short and easy to read, I think I can solve it. The length makes a difference."

When you read a math word problem, how confident are you that you will understand what you are asked to do?

"Pretty confident when it comes to word problems."

What, if anything, is difficult about understanding a math word problem?

"Most of the time it is the strategy if it includes multi-step problems."

Other than the words in a math word problem, what, if anything, is difficult to understand?

"Multi-step problems."

What do you do when you do not understand a word in a math problem?

"I write down all key-words and numbers before I do the problem."

What do you visualize when reading a math word problem?

"If it includes like, items, in real-life items, I think of that. IF it includes multiplication or division, I just do what it says."

What, if anything, helps you visualize a math word problem?

"What is the word, the strategy, I don't know how to explain it. The numbers."

Post-Standardized Math Test Interview Questions

In January, you said you felt "________" when you read a math word problem. What is your feeling now? Do you still have that same reaction?

"Word problems are not that hard anymore, but since we are learning more stuff, I forget a step or overlook an important part in longer word problems."

How confident are you now, when reading a math word problem, that you will understand what you are asked to do?

"Pretty confident if it is readable and I can understand it. Most of the time my answers are right. Readable meaning higher level words."

What is difficult about understanding a math word problem now?

"Not sure. I can understand them now."

Do you use any of the strategies that we practiced in class when you read a word problem? If yes, which ones and how? Please give me an example.

"I stick to what it asks me to do, if I get a chance to use strategies, I will use them."

Data from the study also revealed that the higher confidence levels reported by the students did not always correlate with the correct answers either. In several cases (See example in Figure 7), a student would be able to identify every word in a math word problem except for one. This one word had the ability to change the answer of the problem. Students who could not or did not identify a keyword (domain-specific or general academic vocabulary) that changed the outcome of a problem were still confident in their answers and explanations because they could identify every other part of the problem and still correctly explain how to solve the remaining steps of the problem using number sense abilities. These students were unaware of the existence of the missing step.

One example of this is a word problem asking students to find the mean of a set of data (See example in Figure 7). Students were also explicitly and clearly asked to not include the outlier. Students who did not get the correct answer because they included the outlier were still confident in their answer because they understood the mathematical definition of the word *mean* and they were able to correctly explain all the steps to find the mean of a set of data. However, that one word, *outlier,* changes everything. If students did not identify that word as being important or did not know that word, they did not get the correct answer. If they did not understand the word, *outlier*, they simply did not mention it or ignored it.

No strategies were used by students to use context clues to identify the word and what it meant. This could be that there is limited information to draw from for context clues in a short paragraph that made up a math word problem. For example, there was no paragraph before or after for students to analyze. This supports the claim made by Helman et. Al (2017) that math vocabulary must be explicitly taught during math instruction. However, future research on context clue difference between math and language arts would be an intriguing study.

As with the theme of vocabulary and visualization, an important instructional strategy was to teach students how to break down a problem into steps for the multi-step word problem and confidence theme. Also, teaching students how to include identifying words as a step in breaking down math word problems using a process known as *QUEST!* (A. Bader, personal communication, August 28, 2022) was a suggested teaching strategy to prevent students from missing key words in a math word problem that can change the outcome.

Figure 7

Third Example of a Math Word Problem and Student Response in a Journal Entry

Question 11
Charlie wanted to find the average amount of sprinkles on six cupcakes that he bought for his friends and himself. Without including the outlier, what is the mean amount of sprinkles on the cupcakes with 22, 13, 88, 16, 17, and 12 sprinkles?
Answer:
Incorrect, did not exclude the outlier
Confidence Rating for Answer
confident
Reason or Strategy for your answer
"First, I added them all and then I divided the amount of numbers it had"
Confidence Rating for Reason
confident
Vocabulary
What word in the question helped you figure out this math problem?
What is the definition of this word?
"Mean = average"
Confidence Rating for Vocabulary
confident.
Journal Entry
"I visualized that the words or numbers were combined together at first, and then I imagined dividing by the amount of numbers. Well, the word average did help, it basically meant mean. Plus I am good at doing means too."

During student interviews, many students credited *QUEST!* (Figure 8) with helping them solve multi-step problems (A. Bader, personal communication, August 28, 2022). *QUEST!* Is a process taught to students where this study took place, giving students a series of steps to take when answering a math word problem. While there were no direct links identified in the data to visualizing and using *QUEST!*, Teahen (2015) found that a combination of visualizing and drawing pictures was the most useful strategy for students to use when solving math word problems. Another study found that writing out the process can help students with justifying their own answers for solving a problem in a certain way (MacDonald & Banes, 2017). While *QUEST!* Was not specifically about drawing pictures, it included showing work such as graphs, charts, and writing the steps to find the answer. Students in grades three through eight used this process the entire year at our school during math class to show their work, including all the steps to math word problems. I did not include *QUEST!* In the study's questions, however, after seeing the positive remarks by students when questioned about what helped them with word problems, a future study focusing on the use of *QUEST!* By students on word problems could be beneficial.

Figure 8

An Explanation of QUEST!

Q = Question What is the question? What am I asked to do?
U = Understand What do I already know? What are the key words?
E = Explore What strategy can I use to explore this? ***<u>Draw a picture.</u>*** ***<u>Write a rule, equation or use a formula.</u>*** Look for a pattern. Solve a simpler problem. Estimate. Eliminate. Work Backwards. Make a table. ***<u>Break into Parts.</u>*** Guess and Check. Use an object. Act it out.
S = Solve Apply the strategy and solve the problem. Show the step-by-step solution.
T = Test Did I answer the question? Did I write my answer using a sentence? Does my answer make sense?
! What was difficult? Did I get confused? Where? What did I learn to do next time? How does this relate to the real world?

Vocabulary & Number Sense

The next three action cycles were performed after the week of distance learning leading up to the annual standardized math exam. The main theme that emerged from this data was the struggle many students continue to have with multi-step word problems. All the teaching strategies were geared for students to comprehend the steps they were asked to do, verbally explain what those steps were, and verbally explain how to complete those steps, along with showing their work on paper. Specifically focusing on domain-specific vocabulary instruction had morphed into a daily focus on spiral vocabulary, general academic vocabulary, synonyms, domain-specific vocabulary, and vocabulary that relates to the domain-specific vocabulary of the math taught at that time. An example of one word linking to greater number sense understanding and other domain-specific and general vocabulary is the word, *cube*. Cube is a simple word, only four letters, however, everything that a cube represents is much more complex than just four letters. Knowledge of a definition of the word cube, and knowledge of how to use the word cube in context contains, but is not limited to spiral vocabulary: volume, surface area, six congruent sides, area, perimeter, three dimensional, two dimensional, one dimensional, squared, cubed, etc.

This knowledge of all the vocabulary culminating in *cube* is a clear example of compounding number sense components (such as the word volume being tied to the number sense component of understanding the effect of operation, understanding mathematical properties, facility with various methods, etc.). As a reminder, this compounding of components can cause inadequacies in an educator's instructional strategies concerning EL students (Yang & Lin, 2015) about the breadth and depth of potentially unknown vocabulary (Helman et al., 2017), as well as a lack of using EL teaching strategies (Li, 2018). The possible unknown vocabulary connected to just one word represents several possible unknown number sense abilities as well. It is imperative for educators to understand how and what math vocabulary and number sense abilities compound. Therefore, I created a resource showing the relationships between Minnesota math standards, math vocabulary, and number sense components in Appendix A and in

Appendix B as a compressed version of Appendix A to display number sense components introduced in each grade level (K-6), that I have coined as *Elementary Number Sense.*

Another example of compounding number sense is understanding the effect of operation, better known as math fact fluency in educational settings. Math facts can be thought of as part of an interconnected network (Schwartz, 2023), which are number sense constructs. Math facts, or understanding the effect of operations, are introduced in kindergarten with basic addition and subtraction problems. Understanding the effect of an operation is then compounded with multi-digit addition and subtraction problems, then multiplication and division problems, then multi-digit multiplication and division problems and so forth all the way through elementary school and beyond.

The final two action cycles point to retention of all forms of vocabulary as well as number sense strategies and abilities which continued to be an issue for some students, while other students showed that they retained the vocabulary by using it during class. For the students who were struggling, I observed that their attention span was lacking, possibly because it was close to the summer vacation. Also, many of my observations mention this lack of retention involved fractions. For example:

> “Vocabulary instruction was lacking this week, as stated before. Students' use of number sense strategies was seen in a few of the higher-performing EL students, however, for the rest of the EL students, their knowledge from previous lessons throughout the school year has not been retained as well as I had hoped. Understanding of how to create and use equivalent fractions is an essential number sense strategy that is clearly a topic (along with corresponding vocabulary) that must be taught and retaught frequently throughout the school year.”

The students' responses during interviews about what was difficult about word problems revealed that two students specifically mentioned fractions or division as being difficult. Thus, the difficulties some students showed in retaining math vocabulary may involve vocabulary related to specific number sense abilities, such as fractions and understanding the effect of operations (division).

The data from the final action cycles also showed that the same students who use math vocabulary to explain how to solve problems to their classmates did very well on their assignments. The same students who usually did well on the math journals and with the tests at school were the students who use math vocabulary more often during class. This was evident from the observations, action memos, and students' journal data.

An example of this was one student who spent a lot of time practicing their vocabulary by explaining problems together with their classmates and had a very detailed answer to a word problem involving number sense ability with the word *cube.* The math journal question was: *What is the two-dimensional measurement of a cube with each face of the cube having an area of 4 square inches? What is the one-dimensional measurement of one face of the same cube?* The student's explanation on how to solve this problem was:

> "One face = the flat area on a 3D shape. One dimensional is the perimeter of a shape…I just thought of a cube then decomposed it into one square. Then I tried to look for the length, width, and height by figuring out [2] x [2] is 4. The vocabulary and the information they included in the text helped me visualize."

The student showed their work by drawing a decomposed web of a cube, labeled the measurements, and explained each step of the word problem using domain-specific (cube, square, perimeter), general academic (decompose, flat), and spiral (length, width, height) math vocabulary. The answers to this student's classwork, homework, and tests were also up to the quality of their math journal entries.

Regardless of getting the correct answer to a word problem, if a student was able to identify a specific word in a math word problem and use that word to describe the steps of how to solve part of or all the math word problem, then it shows a correlation between math vocabulary and number sense ability. In many cases, not just with the *outlier* math word problem in Figure 7, students who used vocabulary to explain how to solve a problem were correct in their explanations of the specific group of number sense strategies they were referring to.

I also observed that confidence levels were high for most students during class while students were using structured dialogues and discussing the steps to math word problems. Other students were, however, improving with their use of spiral math vocabulary at the end of the school year. Students were, from what I observed in class and discovered in their journals, more confident in using number sense strategies by knowing the vocabulary. An example that supports the claim that math is a social subject requiring mathematicians (of any age) to produce reasonable arguments to convince other mathematicians of their results is described by Boaler (2015):

> "Most students are now comfortable talking about math and explaining the steps. They are not always correct, but they are trying, making mistakes, and correcting those mistakes all while using more math vocabulary to understand why." And, "Students are still politely correcting other students, using evidence and math vocabulary to explain why."

Most of my research was set up to understand the confidence levels of EL students in relation to vocabulary and number sense ability. However, what I observed, along with the students' responses to journal questions about seeking help from their peers first, showed a conversational confidence that I did not expect to find.

Key instructional suggestions for the theme of vocabulary and number sense are teaching spiral vocabulary that supports new vocabulary and number sense as well as emphasizing the importance of being able to reason and explain each step of a word problem. Giving students the time and space during class to practice discussing spiral math vocabulary along with newly introduced math vocabulary while working on math word problems with classmates was also a suggested instructional strategy to increase student number sense abilities.

Student Growth Data

Collecting and analyzing data from a national test designed to measure student growth throughout the course of the year, known as the Measures of Academic Progress (MAP) test, was another way to measure the effectiveness of teaching upper-elementary EL students math vocabulary during the school year. The MAP test, unlike a standardized test, gives students math problems closer to each student’s ability level based on previous answers. Nation-wide data was only available for the year 2020 for national average growth scores. Comparing student growth scores for students who took part in my study during the 2022-2023 school year with the national average of student growth scores in 2020 was not ideal, however, it did provide some insight and a benchmark for comparison to other students taking the same test.

The mean of the data for student growth nationwide during 2020 was a growth of slightly over eight points from Fall to Spring (NWEA, 2020). The data used to find the mean for the 2020 growth included all students nation-wide (not just EL students) in matching grade-level to my study participants. The mean growth from Fall 2022 to Spring 2023 for the EL students who participated in my study was 10.25 points (See Table 6).

Table 6

Student Growth Data

Fall 2022 Score	Spring 2023 Score	Growth
193	186	-7
219	218	-1
213	216	+3
212	219	+7
213	223	+10
200	210	+10
217	230	+13
221	235	+14
231	246	+15
228	246	+18
220	239	+19
211	233	+22

The mean average growth for EL students in my study during 2023 was thus higher than the average growth for all students in the same grade in the United States during 2020. Observations, action memos, and student journal data revealed that half of the participants in my study showed a greater ability to verbally share information and show their work in written form using math vocabulary. Furthermore, those six students had an average growth of nearly 17 points on the MAP test, over twice the growth of the 2020 national average of eight points. This data further supported the positive correspondence found between effective math vocabulary instruction and number sense ability.

Answers to Specific Research Sub-Questions

Here, I analyze this data to answer each of the four research sub questions, starting with sub question one, "What role does grade-level math vocabulary instruction play in enhancing the confidence level and use of number sense strategies of upper elementary EL students?" Grade-level vocabulary is domain-specific vocabulary for upper-elementary students. It is evident that learning domain-specific vocabulary simultaneously with number sense strategies in grade-level standards using structured dialogues helped the students learn how to explain the steps to a problem and increased the use and ability of number sense strategies but it did not always increase confidence in multi-step word problems. However, increased confidence in knowing a single word and the number sense strategy associated with that one word is evident. The second sub question to be answered is, "What role does spiral math vocabulary instruction play in enhancing the confidence level and use of number sense strategies of upper elementary EL students?" In this study, many students did not show retention of some words from previous years or from lessons earlier in the school year. Using spiral vocabulary from previous years or lessons along with domain-specific vocabulary from current lessons and standards in structured dialogues during whole-group instruction was found to be an effective way to link vocabulary and number sense strategies for EL students. However, as with sub question one, confidence levels did not always match with ability levels because many of the math word problems used for this study were multi-step word problems. Further research concentrating on single-step word problems may produce different results.

Sub questions three and four are, "How do upper-elementary EL students perceive that math vocabulary instruction contributes to their understanding of early number sense strategies?" And "How do upper-elementary EL students perceive that math vocabulary instruction contributes to their understanding of mature number sense strategies?" Students were quite responsive to vocabulary instruction, both early and mature. However, students did not respond well to vocabulary instruction where they were tasked with writing it down. What students responded well to was verbal instruction followed by ample time to practice verbalizing with their classmates.

Considering that there was no definitive boundary for the early and mature number sense constructs, the effectiveness of vocabulary instruction for this study was viewed as identical for both, as students shared their perceptions on vocabulary as a whole. Whether that whole is an entire word problem containing several different types of vocabulary words, or during the whole school year; most students had similar responses to questions about their thoughts on vocabulary instruction. Many students explained that problems became easier when they knew the words, they were able to visualize problems in their mind, and they were able to communicate their thoughts (showing work, explaining steps, asking for help). A few students also mentioned that the structured dialogues they used were helpful tools to refer back to while completing math assignments.

Conclusion: The importance of math vocabulary to increased number sense ability

When combining the data from all three data collection methods to answer the central question of, "What role, if any, does math vocabulary have in increasing upper elementary students' confidence in their understanding and application of number sense strategies?" It was evident that improved math vocabulary increased the students' application and understanding of number sense strategies, but it did not necessarily contribute to an increase in student confidence levels.

In this study, I found that math vocabulary (domain-specific, spiral, synonyms and general academic vocabulary) contributed to greater understanding of number sense components within math word problems. Math vocabulary increases a students' ability to understand and apply number sense strategies, visualize a math word problem, understand and show the steps to a math word problem, and explain the steps being asked to do in a word problem as well as explain how to complete those steps.

Of the methods I used for this study, the more effective instructional method for students to learn new vocabulary was not by using worksheets where students needed to find and copy definitions and math examples, but rather by providing multiple structured dialogues where vocabulary definitions were embedded in context along with domain-specific, spiral, synonym, and general academic vocabulary. This data fits with current research which stated that multiple repetitions of language were the number one method of how students gain language (Haas & Brown, 2019). This allows students to practice explaining how to solve a problem without being required to identify all the steps and new math vocabulary on their own for the first time. The dialogue was also a tool that students can refer to if they get stuck while working independently. Students must also be explicitly taught how to have a conversation with themselves in their mind to visualize math word problems.

Structured dialogues also allow students to be able to share information. Whether they are sharing information with themselves through mental visualization, with their teacher to show competence in solving a math problem during class or on a test, or with another student to either give or receive assistance solving a problem. Sharing information was typically the end goal in any math word problem, where the answer was the information that was sought in a question. Students who had the ability to share information in a structured dialogue, also had a greater ability to verbally share information and recall all forms of vocabulary in unstructured situations. Students able to verbally share information in an unstructured setting had a greater ability to explain their thought process in written form. Students who had a greater ability to share their thought process in written form expressed greater ability to visualize math word problems as well as showing and explaining the steps to their work. Explaining the steps to a problem using math vocabulary showed greater number sense understanding and ability and led to a greater chance at getting the correct answer for a math word problem. Half of the participants showed a greater ability to verbally share information and show their work in written form. Those six students had an average growth of nearly 17 points on the MAP test, over twice the growth of the national average that includes all students. This finding expands the finding that knowledge of math terminology is a predictor of achievement scores on math exams (Ünal et al., 2021) to connect math terminology (math vocabulary) and achievement scores along with number sense ability.

As for confidence levels, many students reported low confidence, even for questions they were able to identify and explain all the steps to. The opposite happened as well, with some students reporting high confidence levels for incorrect answers when they were able to correctly identify and explain only part of a math word problem. The higher confidence levels for incorrect problems were a result of students not identifying all steps, therefore fully believing that they had the correct answer. However, these students were still able to identify vocabulary words and correctly use the number sense strategy associated with that word. This shows a link between vocabulary and number sense ability even if the students missed other parts of a longer word problem.

Uncovering a link between math vocabulary and number sense was the intention of my study and this was accomplished by exploring the effectiveness of increased math vocabulary instruction for EL students' understanding, application of, and confidence in number sense strategies. The implications and recommendations for these findings range from suggestions for teachers in the classroom, teacher educators, policy makers in education, and further research. These implications and recommendations will be introduced and discussed in Chapter 5 to assist readers of my dissertation with putting data from my study into practice.

CHAPTER 5: IMPLICATIONS AND RECOMMENDATIONS

The purpose of my study was to explore if the intentional teaching of grade level and spiral math vocabulary had any impact on upper-elementary EL students' understanding, application of, and confidence in number sense strategies during general education math instruction, specifically when working on math word problems. The participants in my study were 12 EL students in my math class during the 2022-2023 school year. The methods used for data collection were nine weeks of classroom observations followed by eight action cycles, each of two-three weeks in length. Action cycles consisted of observations, student math journal entries, and student interviews.

I made classroom observations as a researcher during whole-group instruction, group work, and individual work. I recorded my observations in the form of action memos and used observation data to improve my teaching practices. Student math journals consisted of one multi step word problem along with confidence ratings and descriptions of students' answers, reasonings, and vocabulary use per journal entry. Students' interviews were conducted once during the beginning of the school year and once near the end of the school year.

Data from my study revealed a link between math vocabulary and number sense ability along with support for the use of structured dialogues for EL students to learn and practice math vocabulary. These structured dialogues may have increased confidence in student use of math vocabulary during the unstructured peer-to-peer mathematical conversations I observed, which in turn, may contribute to an increase in student growth in number sense ability over the course of a school year. This link between math vocabulary, number sense, and structured dialogues is the key to connecting the current literature surrounding Focus on Form (FonF), number sense, and EL teaching strategies. A FonF type called text-enhancement hinders EL students from learning the content (number sense). However, a FonF type using pre-planned tasks, such as structured dialogues, allows students to focus on the content (number sense) while simultaneously practicing domain-specific vocabulary, spiral, and general vocabulary.

The data further showed that the students' ability to visualize a math word problem was also supported by an increased knowledge of math vocabulary, and that the ability to visualize a math word problem increased the students' ability to explain how to solve a math word problem. Data from this study also showed the students' ability to explain the steps to solve a math word problem was significantly correlated with correctly answering a math word problem and performing well on the MAP growth math exam. However, data surrounding the students' confidence were inconsistent with students correctly answering a math word problem, especially math word problems with more than one step, i.e. students who got an answer correct sometimes reported not being confident in their answer, and vice versa.

When looking forward to the future of EL student education, being knowledgeable about current research and best practices is crucial for educators, policy makers, and researchers to provide the best education possible for EL students. Due to increasing numbers of EL students in the American public education system (Kung et al., 2021; Nieto, 2009) and with research in EL education being fairly new compared to other forms of educational research, the expeditious implementation of findings from my study, and others, may prove to be advantageous for districts, schools, teachers, and students. Data from my study concerning the link between math vocabulary and number sense ability could be used or implemented in various ways, including but not limited to practice, policy, and scholarship, which will be discussed in detail throughout this chapter.

Implications and Recommendations for Practice

Key instructional strategies were mentioned throughout Chapter 4 and will be reiterated here to signify their importance. Increasing EL students' knowledge and use of math vocabulary directly impacted the EL students' knowledge and use of number sense constructs embedded in grade-level math standards. Teaching math vocabulary using structured dialogues during instructional time in the general education classroom throughout an entire school year was beneficial for EL students. In contrast, tasking EL students to learn vocabulary by writing down definitions and generating their own sentences using new vocabulary words did not give EL students the best opportunity to learn those words. Structured dialogues could be created by the general education teacher instructing the math class and students may benefit from ample time to practice using math vocabulary in context, fostering peer-to-peer corrective feedback (Ellis, 2016), and increasing the students' ability to explain the steps to solving math word problems.

Using knowledge of number sense components in the classroom to provide differentiated instruction for students who may need additional instruction on number sense components from previous grades could be facilitated by using Appendix A (Scaffolding of Number Sense, Math Vocabulary, and Standards) and Appendix B (Elementary Number Sense Components). A more nuanced understanding how number sense components and math vocabulary compound as students advance through elementary school could minimize misconceptions of EL students and provide educators with pointed instruction to benefit individual students, groups of students, or an entire class in the general education classroom setting.

EL students could also be explicitly taught how to visualize math word problems as well as how to have a conversation with themselves in their mind to help them work through the steps of a math word problem. Using *QUEST!* (See Figure 8), or a similar process, to help students break down the steps to a math word problem could be taught simultaneously with metacognitive strategies for EL students such as having an internal dialogue, visualization, and self-efficacy.

Integrating EL teaching strategies into the general education classroom could help eliminate opportunity gaps experienced by EL students in the math classroom and can also support non-EL students. There was no control group for this study; however, I observed the EL teaching strategies also helped native-English speakers. Many times in the math classroom, all students are learning new words, regardless of being an EL student or not. The removal of students from the general education classroom for EL services could be replaced by integrating EL instruction into whole-group instruction, including math vocabulary instruction using structured dialogues. This approach could be beneficial for classrooms with both EL and non-EL students.

Implications and Recommendations for Standardized Assessments and Policy

Schools and students would benefit if WIDA ACCESS and other tests designed to identify students qualifying for EL services had additional subject-specific sections for domain-specific vocabulary and number sense-oriented questions. Currently, WIDA identifies four modes of communication, which are reading, writing, speaking, and listening (WIDA, 2020). Along with the four modes of communication, WIDA identifies four key language uses: narrate, argue, inform, and explain. While WIDA does mention the presence of all four key language uses across curriculum, English language development standards are only provided for the key language uses of explaining and arguing for math (WIDA, 2020). A student who does not qualify for EL services during language arts instruction may need more assistance during other areas of content instruction, such as math.

Redesigning (or adding separate subject-specific language tests) and readministering subject-specific EL placement tests could help eliminate opportunity and achievement gaps experienced by EL students in and outside of the math classroom. If a student were to be tested for academic language in each content area, educational policy makers could design EL education policy to give individual students help in the areas where it is needed most. This information could then be used by state and federal policymakers to redesign local and national standardized and growth tests that influence policy maker's decisions on school licensure. Standardized math tests should also have the math word problems rewritten in a way that only focuses on number sense abilities, math vocabulary and related academic vocabulary, and to ensure the contexts used for word problems are real-life situations that most students would be familiar with.

In addition, education policy makers and curricula, in each state, should be clearer about what number sense abilities are embedded in each grade level standard as well as a breakdown of all vocabulary used to create the math word problems on standardized math tests (See Appendix A for an example of number sense abilities embedded in grades K-5). A clear representation of what number sense strategies and math vocabulary students are expected to learn and know could benefit the EL student population and their teachers could use this provide targeted instruction to help close the achievement and opportunity gaps.

The expectations and responsibilities of teachers of EL students in the general education classroom could be reevaluated to ensure that the EL students' needs are met. Differentiated instruction for EL students should be done by the general education classroom teacher on top of any push-in or pull-out instruction from an EL teacher. For classroom teachers to be successful with differentiated EL instruction in the general education classroom, institutions training teachers could provide more rigorous training on how to scaffold teaching for EL students in the classroom. Policy makers could also require extra professional development be provided to classroom teachers during a school year for current EL education best practices.

Implications and Recommendations for Scholarship

This classroom action research study suggests several potential areas for further research. An unexpected finding during the interviews was that many students would go to their classmates first for information they did not understand, before they went to the teacher or attempted to find it on their own. This could mean that the structured dialogue process was fostering student confidence and ability in talking about math concepts with their classmates, however, more research in this area is needed. This finding also has implications for classroom management and student agency during instructional time. Further research on EL student confidence and communicating math concepts could explore confidence with structured dialogues, unstructured discussion, confidence in peer-to-peer interactions using math vocabulary vs confidence in student-to-teacher interactions.

Further research on student confidence levels is also recommended for multi-step math word problems. Data from my study suggests that confidence in an answer is correlated with confidence in strategies used and knowledge of math vocabulary. However, when more than one step is required, confidence levels do not correlate to the knowledge displayed. Further research on this topic could begin where my study was completed by focusing on studying confidence levels in one-step word problems and multi-step word problems.

Number sense is still a new concept that needs to be refined. Further research on number sense will help identify how number sense components are structured and compounded in state and national standards. Minnesota Elementary Number Sense Components are introduced in

Appendix A and Appendix B and were created using components from the Approximate, Early, Foundational, and Mature Number Sense (ANS, ENS, FONS, and MNS) constructs. The order and amount in which number sense components are introduced in elementary school may vary from state to state. Further research could use the number sense constructs and components referenced throughout this dissertation as a basepoint to compare to the state and/or national standards being studied in further research.

Conclusion

The career I chose and the reason I chose to study this topic and work with the participants I taught are because they matter to me. The students who I work with deserve the best education possible, and for that to happen more research is needed to fill in any gaps in the ongoing scholarly conversation surrounding EL students' learning math in the general education classroom. Current research surrounding EL student education is robust in many areas, such as metacognition, language acquisition, and general best practices for educators in the classroom. However, there was an opportunity for further research in math vocabulary acquisition and the effect learning math vocabulary has on EL student understanding and use of number sense strategies.

My study contributed to the scholarly conversation surrounding EL student education by identifying a link between math vocabulary knowledge and number sense ability. My study also identified an effective teaching strategy for EL students to acquire vocabulary using structured dialogues. The use of structured dialogues increased student use of math vocabulary in unstructured settings. The resulting unstructured use of math vocabulary by students to explain the steps to solve a math word problem or give peer-to-peer corrective feedback (Ellis, 2016), was a predictor for higher scores on classwork, homework, curriculum exams, and national assessments. Six student participants (out of 12) had an average growth of nearly 17 points on the MAP test, over twice the growth of the national average.

Greater knowledge of math vocabulary increases student application and understanding of number sense strategies but does not necessarily contribute to an increase in student confidence levels. Confidence levels for students were high when relating math vocabulary to a single number sense strategy. Confidence levels for students were low with word problems that had multiple steps. Student ability to explain the steps to a problem using math vocabulary showed greater number sense understanding and ability and led to a greater chance at getting the correct answer for a math word problem.

The scholarly conversation will continue to be told with further research and refined practices in the EL education field of study. My dissertation has identified data showcasing the importance of teaching math vocabulary in general education classrooms along with identifying new questions for further research. EL students, like every other student, deserve the best education possible regardless of the education system they find themselves in. It is my hope that my study benefits at least one, but as many EL students as possible, in the math classroom to overcome any misconceptions along with opportunity and/or achievement gaps through learning about math vocabulary, number sense strategies, and how their own mind learns best.

REFERENCES

Andrews, P., & Sayers, J. (2015). Identifying opportunities for grade one children to acquire foundational number sense: Developing a framework for cross cultural classroom analyses. *Early Childhood Education Journal*, *43*(4), 257-267.

Anghileri, J. (2000). *Teaching number sense*. A&C Black.

Anstrom, K., DiCerbo, P., Butler, F., Katz, A., Millet, J., & Rivera, C. (2010). A review of the literature on academic English: Implications for K-12 English language learners. *Arlington, VA: The George Washington University Center for Equity and Excellence in Education.*

Bandura, A. (1994). Self-efficacy. In V. S. Ramachandran (Ed.), *Encyclopedia of human behavior* (Vol. 4, pp. 71-81). Academic Press.

Barwell, R., Clarkson, P., Halai, A., Kazima, M., Moschkovich, J., Planas, N., Setati-Phakeng, M., Valero, P., & Villavicencio Ubillús, M. (2016). *Mathematics education and language diversity: The 21st ICMI Study*. Springer Nature.

Blachowicz, C. L. Z., Fisher, P. J. L., Ogle, D., & Watts-Taffe, S. (2006). Vocabulary: Questions from the classroom. *Reading Research Quarterly*, *41*(4), 524–539.

Boaler, J. (2015). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages and innovative teaching*. John Wiley & Sons.

Caprara, G. V., Fida, R., Vecchione, M., Del Bove, G., Vecchio, G. M., Barbaranelli, C., & Bandura, A. (2008). Longitudinal analysis of the role of perceived self-efficacy for self-regulated learning in academic continuance and achievement. *Journal of Educational Psychology*, *100*(3), 525.

Cummins, J. (1986). Empowering minority students: A framework for intervention. *Harvard Educational Review*, *56*(1), 18-37.

Cummins, J., & Man, E. Y. F. (2007). Academic language: What is it and how do we acquire it? In J. Cummins, & C. Davison (Eds.), *International handbook of English language teaching* (Vol. 15, pp. 797-810). Springer Science & Business Media.

Danna, R. (2019, July 11). *The spread of Hindu-Arabic numerals in the tradition of European practical mathematics.* Economic History Society. https://ehs.org.uk/the-spread-of-hindu-arabic-numerals-in-the-tradition-of-european-practical-mathematics/

Dweck, C. S. (2006). *Mindset: The new psychology of success*. Random House.

Ebbers, S. M. (2004). *Vocabulary through morphemes: Suffixes, prefixes, and roots for intermediate grades*. Sopris West.

Ellis, R. (2016). Focus on form: A critical review. *Language Teaching Research*, *20*(3), 405-428.

English-language learner. (2013, July 29). The glossary of education reform. Retrieved October 8, 2022 from https://www.edglossary.org/english-language-learner/

Fix, M., & Passel, J. S. (2003). *US immigration: Trends and implications for schools.* Presentation for the National Association for Bilingual Education, NCLB Implementation Institute. New Orleans, LA, January 2003. Retrieved June 28, 2022, from https://files.eric.ed.gov/fulltext/ED474609.pdf

Frymier, A. B., & Houser, M. L. (2000). The teacher-student relationship as an interpersonal relationship. *Communication Education*, *49*(3), 207-219.

Haas, E. M., & Brown, J. E. (2019). *Supporting English learners in the classroom: Best practices for distinguishing language acquisition from learning disabilities*. Teachers College Press.

Helman, L. (2008). English words needed: Creating research-based vocabulary instruction for English learners. In A. E. Farstrup, & S. J. Samuels (Eds.), *What research has to say about vocabulary instruction* (pp. 211-237). International Reading Association.

Helman, L., Cramer, K., Johnston, F., & Bear, D. R. (2017). *Words their way: Vocabulary for elementary mathematics*. Pearson.

Henry, D. L., Baltes, B., & Nistor, N. (2014). Examining the relationship between math scores and English language proficiency. *Journal of Educational Research and Practice, 4*(1), 2.

Herr, K., & Anderson, G. L. (2014). *The action research dissertation: A guide for students and faculty*. Sage publications.

Hinchey, P. H. (2008). *Action research.* Peter Lang.

Hmong in Minnesota. (n.d.). Minnesota historical society. Retrieved October 8, 2022 from https://www.mnhs.org/hmong#:~:text=The%20Hmong%20%E2%80%94%20a%20distinct%20ethnic,concentration%20of%20Hmong%20in%20America.

Howe, C. (2010). Peer dialogue and cognitive development. *Educational dialogues: Understanding and promoting productive interaction*, 32-47.

Jordon, N. C., Kaplan, D., & Hanich, L. B. (2002). Achievement growth in children with learning difficulties in mathematics: Findings of a two-year longitudinal study. *Journal of Educational Psychology*, *94*(3), 586.

Karen history. (n.d.). Karen organization of Minnesota. Retrieved October 8, 2022 from https://www.mnkaren.org/history-culture/karen-history/

Kung, M., Stolz, K., Lin, J., Foster, M. E., Schmitt, S. A., & Purpura, D. J. (2021). The home numeracy environment and measurement of numeracy performance in English and

Spanish in dual language learners. *Topics in Early Childhood Special Education*, *40*(4), 241-252.

Lather, P. (2006). Paradigm proliferation as a good thing to think with: Teaching research in education as a wild profusion. *International Journal of Qualitative Studies in Education*, *19*(1), 35-57.

Lestari, W., & Jailini. (2018, September). Enhancing an ability: Mathematical reasoning through metacognitive strategies. In *Journal of Physics: Conference Series*, *1097*(1), 012117.

Li, G. (2018). Moving toward a diversity plus teacher education: Approaches, challenges, and possibilities in preparing teachers for English language learners. In D. Polly, M. Putman, T. M. Petty, & A. J. Good (Eds.), *Innovative practices in teacher preparation and graduate-level teacher education programs* (pp. 215-236). IGI Global.

MacDonald, L. R., & Banes, L. C. (2017). More than words: Struggling readers' comprehension of word problems. *Journal of Teacher Action Research*, *3*(3), 24-39.

McIntosh, A., Reys, B. J., & Reys, R. E. (1992). A proposed framework for examining basic number sense. *For the Learning of Mathematics*, *12*(3), 2-44.

Minnesota Department of Education. (2022, May). *Mathematics.* https://education.mn.gov/MDE/dse/stds/Math/

Minnesota Department of Education. (n.d.). *English learner education.* https://education.mn.gov/MDE/dse/el/

Moon, K., & Blackman, D. (2014). A guide to understanding social science research for natural scientists. *Conservation Biology*, *28*(5), 1167-1177.

Muhid, A., Amalia, E. R., Hilaliyah, H., Budiana, N., & Wajdi, M. B. N. (2020). The effect of metacognitive strategies implementation on students' reading comprehension achievement. *International Journal of Instruction*, *13*(2), 847-862.

Nagy, W., & Townsend, D. (2012). Words as tools: Learning academic vocabulary as language acquisition. *Reading research quarterly*, *47*(1), 91-108.

National Center for Education Statistics. (May, 2022). English Learners in Public Schools. *Condition of Education*. U.S. Department of Education, Institute of Education Sciences. Retrieved June 28, 2022, from https://nces.ed.gov/programs/coe/indicator/cgf.

National Council of Teachers of English. (2017, September 7). *ELS, ELL, generation 1.5 - why are these terms important?* https://ncte.org/blog/2017/09/esl-ell-generation-1-5-why-are-these-terms-important/

Nelson, G., Hughes Pfannenstiel, K., & Zumeta Edmonds, R. (2020). Examining the alignment of mathematics instructional practices and mathematics vocabulary between core and intervention materials. *Learning Disabilities Research & Practice*, *35*(1), 14-24.

Nieto, D. (2009). A brief history of bilingual education in the United States. *Perspectives on Urban Education*, *6*(1), 61-72.

Northwest Evaluation Association. (2020). *2020 NWEA MAP growth normative data overview.* https://teach.mapnwea.org/impl/MAPGrowthNormativeDataOverview.pdf

Office of the Revisor of Statutes. (2014). *Minnesota session laws - 2014, regular session.* Retrieved June 28, 2022, from https://www.revisor.mn.gov/laws/2014/0/Session+Law/Chapter/272/

Park, J., & Brannon, E. M. (2014). Improving arithmetic performance with number sense training: An investigation of underlying mechanism. *Cognition*, *133*(1), 188-200.

Powell, S. R., Driver, M. K., Roberts, G., & Fall, A.-M. (2017). An analysis of the mathematics vocabulary knowledge of third- and fifth-grade students: Connections to General Vocabulary and Mathematics Computation. *Learning and Individual Differences*, *57*, 22–32. https://doi.org/10.1016/j.lindif.2017.05.011

Rastle, K. (2019). The place of morphology in learning to read in English. *Cortex*, *116*, 45-54.

Riccomini, P. J., Smith, G. W., Hughes, E. M., & Fries, K. M. (2015). The language of mathematics: The importance of teaching and learning mathematical vocabulary. *Reading & Writing Quarterly*, *31*(3), 235-252.

Rizzuto, C. K. (2017). Teachers' perceptions of ELL students: Do their attitudes shape their instruction?. *The Teacher Educator*, *52*(3), 182-202.

Salend, S. J., Garrick Duhaney, L. M., & Montgomery, W. (2002). A comprehensive approach to identifying and addressing issues of disproportionate representation. Remedial and Special Education, 23, 289 – 299. doi:10.1177/07419325020230050401

Sedova, K., Sedlacek, M., Svaricek, R., Majcik, M., Navratilova, J., Drexlerova, A., Kychler, J., Salamounova, Z. (2019). Do those who talk more learn more? The relationship between student classroom talk and student achievement. *Learning and Instruction*, *63*, 101217.

Schleppegrell, M. J. (2007). The linguistic challenges of mathematics teaching and learning: A research review. *Reading & Writing Quarterly*, *23*(2), 139-159.

Schleppegrell, M. J. (2012). Academic language in teaching and learning: Introduction to the special issue. *The Elementary School Journal*, *112*(3), 409-418.

Shim, J. M., & Shur, A. M. (2018). Learning from ELLs' perspectives: Mismatch between ELL and teacher perspectives on ELL learning experiences. *English Language Teaching*, *11*(1), 21-32.

Shockman, E. (2022, August 25). *Minnesota students continue to see pandemic impacts to math and reading.* MPRnews. https://www.mprnews.org/story/2022/08/25/math-and-reading-proficiency-continues-to-drop

Soland, J., Sandilos, L. E. (2021). English language learners, self-efficacy, and the achievement gap: Understanding the relationship between academic and social-emotional growth. *Journal of Education for Students Placed at Risk (JESPAR)*, *26*(1), 20-44.

Stoianov, I., & Zorzi, M. (2012). Emergence of a 'visual number sense' in hierarchical generative models. *Nature Neuroscience*, *15*(2), 194-196.

Teahen, R. J. (2015). *Exploring visualisation as a strategy for improving year 4 & 5 student achievement on Mathematics word problems.* [Master's thesis, Victoria University of Wellington].

Theoharis, G., & O'Toole, J. (2011). Leading inclusive ELL: Social justice leadership for English language learners. *Educational Administration Quarterly*, *47*(4), 646-688.

Ünal, Z. E., Powell, S. R., Özel, S., Scofield, J. E., & Geary, D. C. (2021). Mathematics vocabulary differentially predicts mathematics achievement in eighth grade higher-versus lower-achieving students: Comparisons across two countries. *Learning and Individual Differences*, *92*, 102061.

U.S. Department of Education. (2022). *Every student succeeds act (ESSA).* https://www.ed.gov/essa?src=rn

Walqui, A. (2006). Scaffolding instruction for English language learners: A conceptual framework. *International Journal of Bilingual Education and Bilingualism*, *9*(2), 159-180.

Whitacre, I., Henning, B., & Atabaș, Ş. (2020). Disentangling the research literature on number sense: Three constructs, one name. *Review of Educational Research*, *90*(1), 95-134.

WIDA. (2020). *WIDA English language development standards framework, 2020 edition: Kindergarten–grade 12.* Board of Regents of the University of Wisconsin System. https://wida.wisc.edu/sites/default/files/resource/WIDA-ELD-Standards-Framework-2020.pdf

Yang, D. C., & Lin, Y. C. (2015). Assessing 10-to 11-year-old children’s performance and misconceptions in number sense using a four-tier diagnostic test. *Educational Research*, *57*(4), 368-388.

Zakaria, Z., & Aziz, M. S. A. (2011). Assessing students' performance: The second language (English language) factor. *The International Journal of Educational and Psychological Assessment*, *6*(2), 42-62.

Zviedrite, N., Hodis, J. D., Jahan, F., Gao, H., & Uzicanin, A. (2021). COVID-19-associated school closures and related efforts to sustain education and subsidized meal programs, United States, February 18–June 30, 2020. *PloS One, 16*(9), e0248925.

APPENDIX A: SCAFFOLDING OF NUMBER SENSE, MATH VOCABULARY, AND STANDARDS

The purpose of this appendix is to provide an insight into scaffolding of number sense abilities and the vocabulary words that support them. Students struggling with grade-level standards may be missing or not understanding a number sense ability or word taught to them one or more years prior. Using various research surrounding number sense, along with my own research on number sense and domain-specific math vocabulary, I have created what will be referred to as a scaffolding or flow-chart of Minnesota math standards and the vocabulary and number sense strategies/abilities/constructs that support them. This document is an example of compounding number sense and vocabulary through elementary, not an exactitude. Sixth grade is not included as there is only one more number sense ability introduced: Ordering percents within and among number types.

Superscript “a” denotes “approximate # sense” (Park & Brannon, 2014; Stoianov & Zorzi, 2012)
Superscript “b” denotes “early # sense” (Howell & Kemp, 2010)
Superscript “c” denotes “foundational # sense” (Andrews & Sayers, 2014)
Superscript “d” denotes “mature # sense” (McIntosh, Reys & Reys, 1992)
Superscript “e” denotes refined # sense ability identified by my study

Kindergarten

Standard	# Sense Ability	Domain-specific Vocabulary	General Vocabulary
Understand the relationship between quantities and whole numbers up to 31.	[b] Rote counting/ [c] Systemic counting [c] Number recognition [b] Count from a given # [b] Count backwards [d] Comparing to physical referent	increase decrease Whole number	More, more than Less, less than Order Whole Before, after sequence

	[d] Comparing to mathematical referent [d] Ordering numbers within and among number types [e] Addition & Subtraction operation on whole numbers		
Use objects and pictures to represent situations involving combining and separating.	[b] 1:1 Correspondence [b] Cardinal value [d] Comparing to physical referent [d] Understand the relationship between Addition & subtraction [d] Equivalent numerical forms (decompose/recompose) [e] Addition & Subtraction operation on whole numbers	Sum, plus, add, addition Difference, minus, subtract, subtraction Compose, decompose	Object Put together Taken/take apart
Recognize, create, complete, and extend patterns.	[c] Awareness of number patterns	Skip count	Skip Pattern, repeat Growing, shrinking Shape, color, size
Recognize and sort basic two-and three-dimensional shapes; use them to model	[a] Visual & Tactile Numerosity (ANS) Brain Neurons, Innate	2D: Two Dimensional Square, rectangle, triangle, circle,	Shape, color, size, thick, thin

real-world objects.		trapezoid, hexagon 3D: Three Dimensional Cube, cone, pyramid, cylinder, sphere	
Compare and order objects according to location and measurable attributes.	[a] Visual & Tactile Numerosity (ANS) Brain Neurons, Innate		Length, Size, Weight, and Position vocabulary. Shorter, longer, higher, lower, lighter, heavier, etc.

1st Grade

Standard	# Sense Ability	Domain-specific Vocabulary	General Vocabulary
Count, compare and represent whole numbers up to 120, with an emphasis on groups of tens and ones.	[b] Rote counting [b] Count from a given # [b] Count backwards [b] Cardinal value [d] Comparing to physical referent [d] Comparing to mathematical referent [d] Ordering numbers within and among number types [e] Addition & Subtraction operation on whole numbers [e] Whole number place value	Place value Tens, ones, hundreds increase decrease Whole number Sequence Equal to, not equal to, more than, less than, fewer than, is about, is nearly	More, more than Less, less than Order Whole Before, after Compare, analyze

Use a variety of models and strategies to solve addition and subtraction problems in real-world and mathematical contexts.	[e] Addition & Subtraction operation on whole numbers [c] Simple Arithmetic Competence [e] Make tens [c] Awareness of number patterns [d] Equivalent numerical forms (decompose/recompose)	Part, total (whole) Part + part = whole Compose (put together), decompose (take apart)	Put together, take apart Skip count
Recognize and create patterns; use rules to describe patterns.	[c] Awareness of number patterns [d] Understanding the effect of operations		Repeating, growing, shrinking Recognize, pattern More than, less than
Use number sentences involving addition and subtraction basic facts to represent and solve real-world and mathematical problems; create real-world situations corresponding to number sentences.	[e] Addition & Subtraction operation on whole numbers [c] Simple Arithmetic Competence [d] Recognize reasonableness of calculation [d] Inclination to utilize an efficient representation and/or method [d] Compare to physical/mathematical referent [d] Understanding mathematical properties (commutative property) &	Word problem (True) Equal to (False) Not equal to (False) Inequality Inverse (opposite)	Situation Sentence Missing (blank) total

	(inverses) [d] Understanding the relationship between operations (addition/subtraction)		
Describe characteristics of basic shapes to compose and decompose other objects in various contexts.	[a] Visual & Tactile Numerosity (ANS) Brain Neurons, Innate [d] Compare to physical/mathematical referent	Compose (combine) Decompose Two-dimensional Three-dimensional Triangle, square, rectangle, circle, rectangular prism, cylinder, cone, sphere	Put together Take apart Object, shape
Use basic concepts of measurement in real-world and mathematical situations involving length, time and money.	[d] Compare to physical/mathematical referent [d] Understanding the relationship between operations (addition/subtraction)	Length Amount	Measure (Time related vocabulary) (money related vocabulary)

2nd Grade

Grade Level Standard	# Sense Ability	Domain-specific Vocabulary	General Vocabulary
Compare and represent whole numbers up to 1000 with an emphasis on place value and equality.	[e] Whole number place value [e] Rounding to a specified place value [b] Count from a given number [c] Systemic counting	Equality Place Value Thousands, Hundreds, Tens, Ones Round Decompose	Before After More than Less than

	[b] Counting backwards [d] Ordering numbers within and among number types [d] Understand the relationship between operations [d] Understanding mathematical properties (commutative & associative) [d] Equivalent numerical forms (including decomposition/recomposition) [d] Compare to physical/mathematical referent	Digit Commutative Property Associative Property	
Demonstrate mastery of addition and subtraction basic facts; add and subtract one and two digit numbers in real-world mathematical problems.	[d] Operation on whole numbers [e] (sum/difference) [d] Understanding the relationship between operations [d] Recognize data as exact or approximate [c] Estimate [d] Awareness that solutions may be exact or approximate [d] Compare to	Decompose Expanded notation Partial Sums/Differences Estimate Place value Equality	Graph Tally chart Table

	physical/mathematical referent [d] Multiple representations for numbers (graphical/symbolic) [d] Ability to apply different strategies [d] Ability to select an efficient strategy		
Recognize, create, describe, and use patterns and rules to solve real-world problems and mathematical problems.	[c] Awareness of number patterns [d] Understanding the effect of operations [d] Compare to physical/mathematical referent	Skip count	
Use number sentences involving addition, subtraction, and unknowns to represent and solve real-world and mathematical problems; create real-world situations corresponding to number sentences.	[d] Multiple representations for numbers (graphical/symbolic) [d] Recognize reasonableness of data [d] Understanding the relationship between operations [d] Understanding mathematical properties (inverses) [d] Compare to physical/mathematical referent	Variable Term Equation Inverse (opposite)	Unknown
Identify, describe,	[d] Understanding	Two-dimensional	Edge

and compare basic shapes according to their geometric attributes.	mathematical properties [e](dimensional) [d] Compare to physical/mathematical referent	Three-dimensional (2D and 3D shape names) Face Vertices (corners) Perimeter Area Volume	Side
Understand length as a measurable attribute; use tools to measure length.	[d] Compare to physical/mathematical referent [d] Recognize reasonableness of data [d] Facility with various methods (mental, calculator, paper/pencil)	Unit Segment	Measure Length Size
Use time and money in real-world and mathematical situations.	[d] Ordering numbers within and among number types [d] Place value [e](decimal) [d] Compare to physical/mathematical referent	Quarter (one/fourth) Fraction	Time vocabulary Money vocabulary Combination

3rd Grade

Grade Level Standard	# Sense Ability	Domain-specific Vocabulary	General Vocabulary

Compare and represent whole numbers up to 100,000 with an emphasis on place value and equality.	[c] Systemic counting [e] Whole number place value [e] Rounding to a specified place value [d] Compare to physical/mathematical referent [d] Equivalent numerical forms (including decomposition/recomposition)	Ones, tens, hundreds, thousands, ten thousands Expression/Equation Operation Decompose Round	Compare More Less
Add and subtract multi-digit whole numbers; represent multiplication and division in various ways; solve real-world and mathematical problems using arithmetic.	[e] Whole number place value [d] Understanding the relationship between operations [d] Compare to physical/mathematical referent [d] Understanding mathematical properties (inverses) [d] Operation on whole numbers [e] (sum/difference, product/quotient) [d] Multiple representations of numbers (graphical/symbolic) [d] Awareness that multiple strategies exist:	Inverse Place value Multiplication, division, addition, subtraction Product, quotient, sum, difference	

	Ability to select an efficient strategy [d] Facility with various methods (mental, calculator, paper/pencil)		
Understand meanings and uses of fractions in real-world and mathematical situations.	[d] Sense of relative and absolute magnitude of FRACTIONS: compare to physical/mathematica l referent [d] Ordering numbers within and among number types	part/part/whole Fraction Numerator Denominator “Fractions are division problems”	
Use single-operation input-output rules to represent patterns and relationships and to solve real-world and mathematical problems.	[c] Awareness of number patterns [d] Sense of relative and absolute magnitude of numbers: compare to physical/mathematica l referent [d] Understanding the effect of operations [d] Relationship between number types [d] Ability to create and or invent strategies		pattern
Use number sentences involving multiplication and division basic facts and unknowns to represent and solve real-world and	[d] Multiple representations for numbers (graphical/symbolic) [d] Recognize reasonableness of	Variable Term Equation Inverse (opposite)	Unknown

mathematical problems; create real-world situations corresponding to number sentences.	data [d] Understanding the relationship between operations [d] Understanding mathematical properties (inverses, commutative) [d] Compare to physical/mathematical referent	Commutative	
Use geometric attributes to describe and create shapes in various contexts.	[d] Understand mathematical properties [e](dimensional) [d] Compare to physical/mathematical referent [d] Compare to mathematical referent [d] Multiple representations for numbers	Parallel Perpendicular (shapes vocabulary) Vertices (corners)	
Understand perimeter as a measurable attribute of real-world and mathematical objects. Use various tools to measure distances.	[e] Rounding to a specified place value [d] Understand mathematical properties [e](dimensional) [d] Compare to physical/mathematical referent	Perimeter 2D shape Polygon	
Use time, money and	[d] Ordering numbers	Place value	Temperature

temperature to solve real-world and mathematical problems.	within and among number types [d] Place value [e](decimal) [d] Compare to physical/mathematical referent [d] Awareness that multiple strategies exist		vocabulary Money vocabulary Time vocabulary Elapsed
Collect, organize, display, and interpret data. Use labels and a variety of scales and units in displays.	[d] Multiple representations for numbers (graphical/symbolic) [d] Inclination to review data and result for sensibility	Data Bar graph Frequency table Picture graphs Number Line plots	Label

4th Grade

Grade Level Standard	# Sense Ability	Domain-specific Vocabulary	General Vocabulary
Demonstrate mastery of multiplication and division basic facts; multiply multi-digit numbers; solve real-world and mathematical problems using arithmetic.	[d] Compare to physical/mathematical referent [d] Understand the effect of operations [d] Recognize that data may be exact or approximate [c] Estimation [e] Whole number place value	Place value Algorithm Multi-step Formula	

	[e] Rounding to a specified place value [d] Comparison to benchmarks [d] Recognize reasonableness of calculation [d] Understanding mathematical properties (identities, commutative, associative, distributive, inverse) [e] Ability to compound multiple strategies [d] Facility with various methods (mental, calculator, paper/pencil)		
Represent and compare fractions and decimals in real-world and mathematical situations; use place value to understand how decimals represent quantities.	[d] Compare to physical/mathematical referent [d] Multiple representations for numbers [d] Ability to create and/or invent strategies [d] Ordering number within and among number types [d] Place value [e](decimal) & (whole number) [e] Rounding to a	Equivalent Fraction, improper fraction Numerator, denominator Mixed number Tenths, hundredths, thousandths Ones, tens, hundreds, thousands, ten thousands	

	specified place value		
Use input-output tables and charts to represent patterns and relationships and to solve real-world and mathematical problems.	[c] Awareness of number patterns [d] Compare to physical/mathematical referent [d] Multiple representations for numbers [d] Ability to create and/or invent strategies [d] Understand the effect of operations		
Use number sentences involving multiplication, division and unknowns to represent and solve real-world and mathematical problems; create real-world situations corresponding to number sentences.	[d] Multiple representations for numbers (graphical/symbolic) [d] Recognize reasonableness of data [d] Understanding the relationship between operations [d] Understanding mathematical properties (inverses, commutative) [d] Compare to physical/mathematical referent	Variable Term Equation Inverse (opposite) Commutative Equal (true)	
Name, describe, classify and sketch polygons.	[d] Understanding mathematical properties	Polygon vocabulary	

	[e] (dimensional)		
Understand angle and area as measurable attributes of real-world and mathematical objects. Use various tools to measure angles and areas.	[d] Understanding mathematical properties [e] (dimensional) [d] Facility with various methods (mental, calculator, paper/pencil) [d] Ordering numbers within and among number types [d] Equivalent numerical forms (including decomposition/recomposition) [d] Compare to physical/mathematical referent	Angles Acute, obtuse, right (90) Area, length, width Decompose, recompose Square units Two-dimensional Formula	
Use translations, reflections and rotations to establish congruency and understand symmetries.	[a] Visual & Tactile Numerosity (ANS) Brain Neurons, Innate [d] Compare to mathematical referent [d] Understanding mathematical properties (dimensional)	Translation Reflection Rotation Congruent Symmetrical Clockwise Counterclockwise Coordinate plane X-axis, y-axis Coordinates	Slide Flip
Collect, organize, display and interpret data, including data collected over a period of time and data represented by fractions and decimals.	[d] Multiple representations for numbers (graphical/symbolic) [d] Inclination to review data and result for sensibility	Data Data Bar graph table	spreadsheet

		Venn Diagram Timeline	

5th Grade

Grade Level Standard	# Sense Ability	Domain-specific Vocabulary	General Vocabulary
Divide multi-digit numbers; solve real-world and mathematical problems using arithmetic.	[d] Place value [d] Equivalent numerical forms (including decomposition/recomposition) [d] Understanding the relationship between operations [d] Compare to physical/mathematical referent [d] Understanding the relationship between problem context and the necessary computation: Awareness that data/solutions may be exact or approximate [d] Inclination to review data and result for sensibility [d] Understanding mathematical properties (identities, commutative, associative, distributive, inverse)	Fractions, improper fractions Mixed numbers Remainder Divide, quotient Estimate Inverse	opposite

	[d] Facility with various methods (mental, calculator, paper/pencil)		
Read, write, represent and compare fractions and decimals; recognize and write equivalent fractions; convert between fractions and decimals; use fractions and decimals in real-world and mathematical situations.	[d] Place value [e](decimal) & (whole number) [e] Decimal number after & number before [d] Ordering numbers within and among number types [e] Rounding to a specified place value	From tenths to millionths From ones to millions Equivalent, unequal Fractions, improper fractions Mixed numbers	Before After Greater than, less than
Add and subtract fractions, mixed numbers and decimals to solve real-world and mathematical problems.	[d] Understanding the relationship between operations [d] Place value [e](decimal) & (whole number) [d] Relationship between number types [d] Multiple representations for numbers [d] Understanding the relationship between problem context and the necessary computation: Awareness that data/solutions may be exact or approximate	Add, subtract Sum, difference Numerator, denominator Equivalent, greater than, less than	

	[d] Operating on fractions/decimals		
Recognize and represent patterns of change; use patterns, tables, graphs and rules to solve real-world and mathematical problems.	[d] Understanding the relationship between operations [c] Awareness of number patterns [d] Awareness that multiple strategies exist: ability to apply different strategies [d] Understand mathematical properties	Ordered pairs Integer Graph Coordinate plane (and supporting vocabulary)	rule
Use properties of arithmetic to generate equivalent numerical expressions and evaluate expressions involving whole numbers.	[d] Understanding mathematical properties (identities, commutative, associative, distributive, inverse) [d] Understanding the relationship between operations	Commutative Associative Distributive Expression	
Understand and interpret equations and inequalities involving variables and whole numbers, and use them to represent and solve real-world and mathematical problems.	[d] Understanding mathematical properties (identities, commutative, associative, distributive, inverse) [d] Understanding the relationship between operations [d] Inclination to review data and result for sensibility	Equation Inequality Variable Expression Evaluate Formula	True or false

	[d] Compare to physical/ mathematical referent [d] Ability to select an efficient strategy		
Describe, classify, and draw representations of 3D figures.	[d] Understanding mathematical properties [e](dimensional) [d] Equivalent numerical forms (including decomposition/recomposition) [d] Compare to physical/ mathematical referent	Formula Three Dimensional Net Decompose Two-dimensional (2D and 3D shape names) Face Vertices (corners) Perimeter Area Volume	
Determine the area of triangles and quadrilaterals; determine the surface area and volume of rectangular prisms in various contexts.	[d] Understanding mathematical properties [e](dimensional) [d] Facility with various methods (mental, calculator, paper/pencil) [d] Ordering numbers within and among number types [d] Equivalent numerical forms (including decomposition/recomposition) [d] Compare to physical/ mathematical referent	Area Quadrilaterals, parallelograms Triangle Surface area Volume 3D shapes vocabulary Decompose Cubic unit Formula Variable 1D 2D 3D	

	[d] Ability to create and/or invent strategies, ability to apply different strategies		
Display and interpret data; determine mean, median and range.	[d] Ordering numbers within and among number types [d] Facility with various methods (mental, calculator, paper/pencil) [d] Compare to physical/ mathematical referent [d] Understanding mathematical properties [d] Understanding the relationship between operations [d] Inclination to review data and result for sensibility [c] Awareness of number patterns [d] Place value [e](decimal) & (whole number) [d] Operating on fractions/decimals	Mean, Median, Mode, Range	Data, spreadsheet

APPENDIX B: ELEMENTARY NUMBER SENSE COMPONENTS

Updated table with number sense components based on previous & current number sense research, this study, and the Minnesota Department of Education K-6 mathematics standards.

Natural Components 1) Visual & Tactile (brain neurons, innate)
Authority Components Foster and strengthen to support continued learning in the mathematics classroom every lesson, every grade level. **1) Ability to describe thinking process** **2) Recognize reasonableness of data** **3) Recognize reasonableness of calculation** **4) Use a system of mathematical and personal benchmarks**
Components introduced in Kindergarten 1) Rote/systemic counting 2) Number recognition 3) Count backwards 4) Count from a given number 5) Sense of relative and absolute magnitude of **whole numbers**: Comparing to physical & mathematical referent 6) Ordering whole numbers within and among number types 7) Understanding the effect of operation on whole numbers (addition & subtraction) 8) 1:1 Correspondence 9) Cardinal value 10) Understanding the relationship between addition and subtraction 11) Awareness of number patterns 12) Equivalent numerical forms (decompose/recompose)
Components introduced in 1st Grade <u>These components are compounded to components introduced in kindergarten.</u> 1) Whole number place value (1, 10, 100) 2) Equivalent numerical forms 3) Making "10s" 4) Inclination to review: Recognize reasonableness of data/calculation 5) Inclination to utilize an efficient representation and/or method 6) Understanding mathematical properties (commutative & inverses)
Components introduced in 2nd Grade <u>These components are compounded to components introduced in kindergarten & 1st grade.</u> 1) Whole number place value (1, 10, 100 + **1000**) 2) Rounding to a specified place value 3) Understanding mathematical properties (commutative, inverses, & **associative**)

4) Recognize data as exact or approximate
5) Awareness that solutions may be exact or approximate
6) Multiple representations for numbers (graphical/symbolic)
7) Ability to apply different strategies
8) Ability to select an efficient strategy
9) Facility with various methods (mental, calculator, paper/pencil)
10) Place value: decimal (money)

Components introduced in 3rd Grade

<u>These components are compounded to components introduced in kindergarten, 1st grade, & 2nd grade.</u>

1) Whole number place value (1, 10, 100, 1,000, + **10,000**)
2) Understanding the effect of operation on whole numbers (**multiplication & division**)
3) Sense of relative and absolute magnitude of **FRACTIONS**: Comparing to physical & mathematical referent
4) Ordering **FRACTIONS** within and among number types
5) Ability to create and or invent strategies
6) Place value: decimal (**tenth, hundredth**)
7) Understanding mathematical properties (**distributive**)

Components introduced in 4th Grade

<u>These components are compounded to components introduced in kindergarten, 1st grade, 2nd grade, & 3rd grade.</u>

1) Ability to compound multiple strategies
2) Ordering **mixed numbers & improper fractions** within and among number types
3) Place value: decimal (**thousandths**)
4) Rounding to a specified place value: **decimals**
5) Relationship between number types
6) Understanding mathematical properties (dimensional): **1D & 2D**

Components introduced in 5th Grade

<u>These components are compounded to components introduced in kindergarten, 1st grade, 2nd grade, 3rd grade, & 4th grade.</u>

1) Place value: decimal (**ten thousandths, hundred thousandths, millionths**)
2) Whole number place value (**hundred thousand, million**)
3) Understanding mathematical properties (dimensional): **3D**
4) Operating on fractions/decimals

Components introduced in 6th Grade

<u>This component is compounded to components introduced in kindergarten, 1st grade, 2nd grade, 3rd grade, 4th grade, & 5th grade.</u>

1) Ordering **percents** within and among number types