Fern or Fractal... Or Both?

By **Christina Babcock**

Advisor **Sarah Jahn**

Concordia University·Saint Paul

Introduction A fractal is a series of self similar sets. Michael Barnsley, a British Mathematician who inventented the modern idea of fractals worked with fractal image compression to create a fern fractal using iterated function systems. This fern is now known as the Barnsley Fern and was developed from a Black Spleewort Fern. The fern uses four different affine transformations which develop each part of the fern. The study of this fern and how the iterated fuction system used to create it allowed me to create a fractal shell using iterated fuction systems and the concepts pulled from the Barnsley Fern.

Research Methods Using many sources, including Fractals Everywhere by Michael Barnsley himself, to understanding the inerworkings of the Barnsley Fern.

Results There are four different affine transformations that make up the fern and they are chosen randomly by the Python code. The $f_1(x, y)$ equation brings the point to the bottom of the stem of the fern as denoted in figure 1. This happens 1% of the time. The $f_2(x, y)$ equation generates the curve of the fern seen in figure 2 happening 85% of the time. The $f_3(x, y)$ equation brings the point to the bottom leaf of the left side of the fern seen in figure 3, 7% of the time. And finally, the $f_4(x, y)$ equation brings the point to the bottom leaf of the right side of the fern, also 7% of the time. For the last two equations, if the x value being plugged in is positive, the new point will be on the top half of the leaf and if x is negative, the new point will be on the bottom half of the leaf.

$$f_1(x, y) = \begin{bmatrix} 0 & 0 \\ 0 & 0.16 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$f_2(x, y) = \begin{bmatrix} 0.85 & 0.04 \\ -0.04 & 0.85 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 1.6 \end{bmatrix}$$

$$f_3(x, y) = \begin{bmatrix} 0.20 & -0.26 \\ 0.23 & 0.22 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 1.6 \end{bmatrix}$$

$$f_4(x, y) = \begin{bmatrix} -0.15 & 0.28 \\ 0.26 & 0.24 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 0.44 \end{bmatrix}$$

Figure 1

Figure 2

Figure 3

Figure 4

Figure 5

Discussion After learning and discovering the fundamentals of the Barnsley Fern. I was able to create my own rendition of the fern. Using new rotations and scaling values, my adaptation of the fern curves more and has thinner, more upright leaves.

I was also able to take what I learned with the fern calculations and create a fractal shell using an iterated function system with three different randomly chosen equations. The $f_1(s, t)$ equation in the shell brings the point to somewhere in the curve of the first 15 degrees of the spiral seen in figure 7. This piece is chosen 3% of the time. The $f_2(s, t)$ equation brings the point to different spikes around the shell and makes the shading seen in figure 2, also chosen 3% of the time. When the "t" value plugged in to $f_2(s, t)$ is a whole number, the point will be on the spike. If the "t" value is not an integer, it will create shading. The $f_3(s, t)$ equation is the piece that rotates and creates the spiral shown in figure 3. This final piece is chosen 94% of the time.

$$f_1(s, t) = \begin{bmatrix} 0 & 0.042 \\ 0 & -0.533 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix} + \begin{bmatrix} 0 \\ 250 \end{bmatrix}$$

$$f_2(s, t) = \begin{bmatrix} 0.968 & 0 \\ 0 & -0.042 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix}$$

$$f_3(s, t) = \begin{bmatrix} 0.968 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix} + \begin{bmatrix} 0 \\ 15 \end{bmatrix}$$

Figure 5

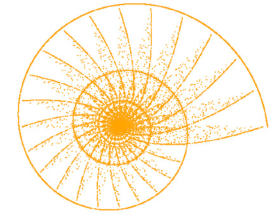

Figure 6

Figure 7

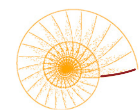

Figure 8

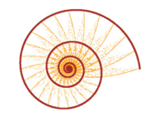

Figure 9

See the shell grow here

Conclusion Using what I learned about the Barnsley Fern, iterated function systems, fractals, and coding in Python, I was able to create my own shell fractal. The shell fractal is self similar, has repeating parts and was made from an iterated function system, all the things that also made up the Barnsley Fern.

Refrences

Barnsley, M. F., et al. "Solution of an Inverse Problem for Fractals and Other Sets." Proceedings of the National Academy of Sciences of the United States of America, vol. 83, no. 7, 1986, pp. 1975–1977. JSTOR, www.jstor.org/stable/27120. Accessed 13 Apr. 2020.

Diaconis, Persi, and Mehrdad Shahshahani. "Products of Random Matrices and Computer Image Generation." Random Matrices and Their Applications Contemporary Mathematics, 1986, pp. 173–182., doi:10.1090/conm/050/841091.

Barnsley, Michael F. "Fractal Image Compression." Notices of the AMS, vol. 46, 1996, pp. 657–662.

Barnsley, M. F. Fractals Everywhere. Dover Publications, 2012.

Garousi, Mehrdad. "The Postmodern Beauty of Fractals." Leonardo, vol. 45, no. 1, 2012, pp. 26–6., www.jstor.org/stable/41421791. Accessed 13 Apr. 2020.

Zobitz, Jennifer. "FRACTALS: MATHEMATICAL MONSTERS." Pi Mu Epsilon Journal, vol. 8, no. 7, 1987, pp. 425–440. JSTOR, www.jstor.org/stable/24337748. Accessed 13 Apr. 2020.

Barnsley, Michael F., and John H. Elton. "A New Class of Markov Processes for Image Encoding." Advances in Applied Probability, vol. 20, no. 1, 1988, pp. 14–32. JSTOR, www.jstor.org/stable/1427268. Accessed 13 Apr. 2020.