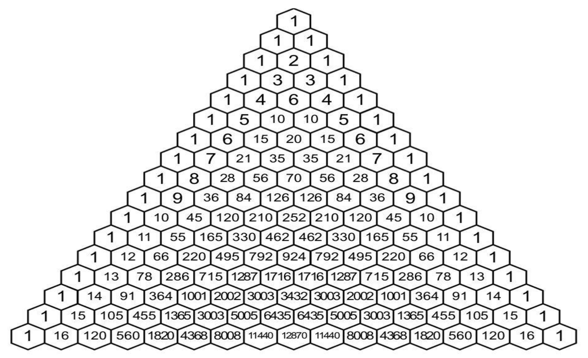
$$_nC_k = \frac{n!}{k!\,(n-k)!}$$



Discovering Pascal's Triangle

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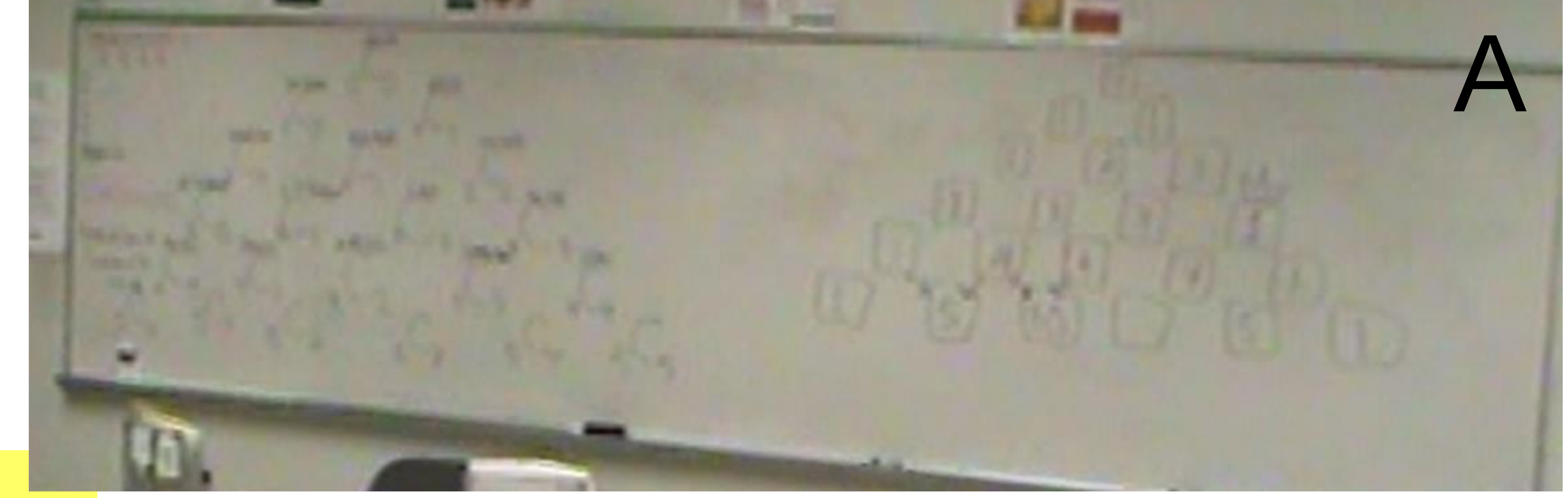

A

Research Question

How can Bruner's theory on discovery learning and the concept of constructivism motivate my differentiated learners to understand Pascal's Triangle and enhance their understanding of combinations?

Introduction

Discovering Pascal's triangle is a lesson that focuses on patterns and promoted student active engagement as students lead the class through their own discovery lesson. As they learned how to use combinations in their own lives, they found curiosity and reason to look further into the topic, which led to the set up for more advanced mathematical thinking as patterns in Pascal's triangle. My students are diverse and differentiated but discovering Pascal' triangle on their own brought them together through classroom leadership, independence, and teamwork.

My lesson plan addresses the research question as it is set up to engage students in active learning through ways of discovery. This is methodically reasonable for my students because they are ones to groan at basic lectures and worksheets, so a pedagogical change was needed. By starting my students with tree diagrams and real-life combinations problems, my students took hold of the combinations topic and dove into the discovery lesson of Pascal's triangle.

Results

As a result, the students were able to find the pattern in the n values and in the addition patterns. Students calculated how many ways there are to chose r items from n distinct items without replacement using formulas, symbols, and common sense of finding all possible outcomes. Students seemed to prefer the addition pattern and the pattern of natural numbers in the second diagonal. As an exit slip, students were able to recreate a copy of Pascal's triangle and calculate more of the triangle than we did as a class.

In result of this lesson, students were able to take leadership within the classroom and discover patterns of Pascal's triangle on their own. Students were able to complete their exit slips to a higher than normal completion rate. About 80% of the students successfully completed the assessment and could comprehend the patterns within Pascal's triangle.

Research Methods

With only prior knowledge in tree diagrams, my students began to practice calculation values of combinations for real world problems. After they noticed a similar outcome, students were introduced to the combination formula. I set up the beginning of Pascal's triangle and called students up to help me rebuild the triangle. After the beginning of the triangle was built, students noticed patterns; first the diagonal ones, then the diagonal natural numbers, and eventually the pattern of addition found from previous rows.

What Did I Do?

My students were initially taught about tree diagrams and the counting principle before we dove into the topic of real-life combinations, scenarios where order does not matter. My differentiated learners (such as the English language learners, struggling readers, students challenged with math, and other disabilities) learn best from real life situations they can relate to, so I began with teaching word problems that represent combinations to entice student interest in the topic of combinations.

What Did the Students Do?

After the students discovered the general pattern of combinations, I introduced the combination formula as a shortcut to calculate the number of ways a scenario can happen. Once the students had adequate practice with the combination formula, I drew the first three rows of Pascal's triangle on the whiteboard in the form of unsolved combinations and assigned individual students to calculate the values (attached video of teaching in References & Links). Students plugged in values (1-3) into the formula for Pascal's to get their answer as some students checked if their answer was correct with me before they wrote their answer in the empty squares to the right (see picture A - whiteboard above or in video). After the first three rows were calculated we discussed the result patterns from "choosing one" and "choosing zero." Next, I had students complete the 4^{th} & 5^{th} row and begin to look for patterns. The first pattern discovered by a student was the exterior 1's, the next pattern found was the string of natural numbers in the first diagonal. When students were prompted to guess the 6^{th} row, there was disagreement as some believe the pattern in the diagonal should go up by 3 (3, 6, 9), so we turned to the combinations formula to solve for the values in the 6^{th} row. Student then discovered the diagonal values are 3, 6, 10 and so on. After the 6^{th} row was found, students noticed the pattern of addition coming from the two values in the previous row.

What Did I Learn?

From this teaching experience, I learned that differentiated learners can excel from discovery lessons that flow from Bruner's theory of discovery learning. I also learned that diverse students excel using constructivism-based opportunities and that calling students to build the triangle gave them ownership of the discovery, which promoted active engagement among students.

Conclusions

In conclusion, the discovery of Pascal's Triangle is a great lesson for students because it gives them a glimpse into the history of mathematics, patterns in upper level math topics, and connections between real life situations using combinations and more abstract math concepts, such as Pascal's triangle.

In application, I can use this research to promote active pattern learning in upper level mathematics classrooms. The discovery opportunities I provided to my students not only applied constructivism and taught me about the learning needs of my students, but it also gave the students control over their learning and built leadership across the classroom as students worked together to discuss ideas and patterns.

As the assessment for Pascal's triangle was brief, the students were still able to perform and assess well. Less than 5% of students in the classroom chose to not participate, and approximately 8% of them were not able to complete the given triangle because of time constraints in the classroom. Many of the students were able to fully calculate and fill out the given assessment, except for a few absent students.

Further Research

In further research, Bruner's theory of discovery learning, this lesson is weak in the sense of Enactive learning, as only some students are moving and writing on the board because of space limitations and time constraints. The lesson does meet areas in Iconic and Symbolic learning according to Bruner's theory.

The theory of constructivism enhances my understanding of my students' need for active engagement in the classroom. As "people learn by using what they know to construct new understandings... [so] all learning involves transfer that is based on previous experiences and prior knowledge" (*How People Learn,* pages 68, 236). By aiding my students in discovery learning and letting the students take control of the lesson and the learning environment, my students were able to excel in the topic of combinations.

Discussion Topics

Have the students been exposed to many patterns?

How could I have incorporated a discovery of the diagonal patterns found in Pascal's triangle, beyond the natural numbers?

In the future, I could relate this to the Fibonacci sequence.

Other: Students were not as interested in a post assignment for discovering Sierpinski's triangle, as the students were not able to find an "easy" pattern for this assignment, such as addition.

References & Links

https://www.simplypsychology.org/bruner.html, http://ptri1.tripod.com/,
https://www.asa3.org/ASA/education/teach/active.htm#constructivism

Link to Lesson Plan:
https://docs.google.com/document/d/1gdipZLoK93EvHE_Lj2lejV24KBL8n0_wbpu43WciECY/edit?usp=sharing

Link to Teaching Video(s):
https://drive.google.com/drive/folders/13giQ00p3VP4fXzgx0ajvnDI53GeznrGK?usp=sharing